Monetary Policy in Estimated Models of Small Open Versus Closed Economies

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Abstract

This paper develops and estimates a quantitative dynamic-optimizing model of a small open economy (SOE) with domestic and import price stickiness and capital-adjustment costs. A monetary policy rule allows the central bank to systematically manage the nominal interest rate in response to deviations of inflation, output, and money growth from their steady-state levels. The structural parameters of the SOE model, as well as those of a sticky-price model for a closed economy (CE), are estimated econometrically using data from Canada and the United States and a maximum-likelihood procedure with a Kalman filter. Estimation results show that the SOE and CE models lead to similar estimates for the Canadian economy. Furthermore, the effects of monetary policy shocks, and of other domestic shocks, generated in the SOE model are isomorphic to those generated in the CE model. Nevertheless, the forecast-error decomposition shows that the importance of domestic demand shocks is reduced by the introduction of foreign shocks.

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1. Introduction

In recent years, an extensive literature has developed that considers new open-economy models based on microeconomic foundations and including nominal rigidities.¹ These models have been used to explore many issues not addressed in the closed-economy (CE) framework, such as the persistence of real and nominal exchange rates (Kollmann 2001; Chari, Kehoe, and McGrattan 2002) and exchange rate pass-through (Devereux and Engel 2002; Smets and Wouters 2002). Nevertheless, almost all of these studies use calibrated, rather than estimated, models to achieve their goals. Despite the increasing number of papers that elaborate different types of small open-economy (SOE) models, their structural parameters are rarely estimated.

In a recent literature search, Ghironi (2000), Smets and Wouters (2002), and Bergin (2003) were the only authors found who estimate some of the structural parameters of a SOE model with nominal rigidities. They each use a different estimation procedure. Ghironi (2000) uses a non-linear least-squares method at the single-equation level to estimate the structural parameters of a SOE model using data from Canada and the United States. The model is then used to show how a shock to the U.S. economy is transmitted to Canada. Smets and Wouters (2002) estimate only the degree of domestic and import price stickiness using data from the euro area and the United States. Their method consists of minimizing the difference between the empirical and theoretical impulse responses to monetary policy and exchange rate shocks. On the other hand, Bergin (2003) uses a maximum-likelihood procedure to estimate the structural parameters of a SOE model using data from Australia, Canada, and the United Kingdom. Using this procedure, he estimates and tests a SOE model with monetary shocks and nominal rigidities. He concludes that the results offer mixed support for his estimated model.

This paper aims to estimate and simulate the structural parameters of SOE and CE models for the Canadian economy, and compares the effects of monetary policy shocks generated in both models. This comparison allows us to determine whether a CE framework is rele-

¹Lane (2001) and Bowman and Doyle (2002) give detailed surveys of this literature.
vant and useful to estimate and simulate such a dynamic general-equilibrium model, even though Canada is a small open economy. The paper first develops a SOE model in which all economic agents exhibit optimizing behaviour and domestic and import prices are sticky. The economy is small because it takes the world interest rate and price level as given. Domestic producing and importing firms are monopolistically competitive with staggered price settings, à la Calvo (1983). Price stickiness in the import sector implies an imperfect pass-through of changes in the real exchange rate and the foreign output price on import prices in the domestic economy.

It is also assumed that domestic households have access to incomplete international financial markets, but the price they must pay is increasing in the foreign-debt-to-output ratio. Thus, the risk-premium term, which reflects departures from uncovered interest parity, is endogenous. This assumption implies a stationary steady state for consumption and net foreign bonds, and allows equations that describe equilibrium in a stochastic model to be derived. In contrast, Bergin (2003) allows the presence of a random walk in equilibrium dynamics, and Ghironi (2000) and Smets and Wouters (2002) use a Blanchard-Yaari-type overlapping generation model to derive a stationary steady state. Following Ireland (2001, 2003), a central bank’s behaviour is described by a monetary policy rule that adjusts the short-term nominal interest rate in response to inflation, output, and money growth.\footnote{Originally, it was assumed that the Bank of Canada also responds to real exchange rate deviations, but estimates of its coefficient are too small and statistically insignificant, so it is omitted from the final rule.} It is a modified Taylor-type rule that ensures equilibrium determinacy as long as the sum of inflation and money-growth coefficients exceeds one. Bergin (2003), however, assumes that the monetary authority follows a money-supply rule.

In the presence of nominal frictions, monetary policy can affect real variables in both closed- and open-economy models. Nevertheless, its transmission mechanism in an open economy generally differs from that in a closed economy. In particular, if the nominal interest rate rises in response to an increase in inflation, the real interest changes in both models. In a closed economy, an increase in the real interest rate leads to a decrease in consumption due to the intertemporal substitution effect. In an open economy, however, a higher real
interest rate leads not only to reduced consumption through intertemporal substitution, but also to an appreciation of the real exchange rate and an improvement in the terms of trade. The appreciation of the real exchange rate and the improvement in the terms of trade tend to increase consumption through an expenditure-switching effect between domestic and imported goods. Thus, the intertemporal substitution and expenditure-switching effects may affect aggregate demand in opposite directions following a change in the real interest rate. Nevertheless, Clarida, Galí, and Gertler (2001), who derive the optimal monetary policy in a SOE model with sticky prices, argue that the problem of monetary policy for the small open economy is isomorphic to that of a closed economy and that all qualitative results obtained in the latter extend to the former.

Following Clarida, Galí, and Gertler’s (2001) argument, we econometrically estimate, for the Canadian economy, the structural parameters of a SOE model as well as those of a CE model using a maximum-likelihood procedure with a Kalman filter. The closed economy is very similar to the sticky-price models estimated by Ireland (2001) for the U.S. economy and by Dib (2002) for the Canadian economy. The estimation procedure is frequently used to estimate CE models (for example, Ireland 2001, 2003; Dib 2002, 2003).

The results show that the estimates of the structural parameters are very similar in the SOE and CE models. For example, the degree of domestic and import price stickiness in the SOE model is estimated to be similar to that estimated in the closed economy. In addition, the estimated values of the coefficients of the monetary policy rule are almost all statistically equal in both economies. Based on the estimated values for the SOE and CE models, we simulate both models to analyze and compare the impact of monetary policy, technology, and world nominal interest rate shocks on some macroeconomic variables. The impulse responses to monetary policy shocks, and to other domestic shocks, are isomorphic in both simulated models. Nevertheless, the forecast-error decomposition shows that the importance of domestic demand shocks is greatly reduced by the introduction of foreign shocks. The expenditure-switching effect is estimated to be small, which leads to only a marginal discrepancy between the responses of consumption to a monetary policy shock in small open and closed economies. This finding empirically supports the view that the effects of mone-
tary policy in a SOE model are qualitatively similar to those in a closed economy.

Furthermore, the SOE model is able to generate high volatility and persistent output, nominal interest rates, and inflation. It fails, however, to reproduce high volatility and persistent real exchange rates, as observed in the data. To reproduce such volatility and persistence, import prices must remain unchanged for at least 3.5 years. This finding is consistent with the main finding of Chari, Kehoe, and McGrattan (2002). The forecast variance error decomposition shows that world nominal interest rate shocks account for more than 85 per cent of real exchange rate fluctuations in the short and long terms.

This paper is organized as follows. Section 2 develops a theoretical SOE model. Section 3 discusses the data and the procedures of calibration and estimation. Section 4 reports and discusses the empirical results. Section 5 concludes.

2. The Model

This section develops a structural dynamic model for a small open economy, following in particular Dib (2002), Ireland (2001, 2003), and Kollmann (2001, 2002), with domestic and import price stickiness. There are five agents: a representative household, a continuum of domestic producers and importers indexed by $j \in [0,1]$, an aggregator, and a monetary authority. Domestic households have access to incomplete international financial markets, but they must pay a risk premium that is increasing in the foreign-debt-to-output ratio.

Domestic producers and importers are monopolistically competitive with staggered price settings, à la Calvo (1983). Each producer produces a distinct domestic-intermediate good using capital and labour as inputs. The produced good is divided between home market use and exports, and its producer cannot price-discriminate between the two markets. The importers import a homogeneous good produced abroad to produce a differentiated imported-intermediate good for home market use. The aggregator uses the domestic- and imported-intermediate goods to produce domestic- and imported-composite goods, which it turns into a final good using a constant elasticity of substitution (CES) production technology. The final good is divided between home consumption and investment. The economy is small
because home agents take the world nominal interest rate and price level as given.

2.1 Households

The representative household derives utility from consumption, $c_t$; real balances, $M_t/p_t$; and leisure, $1 - h_t$. Its preferences are described by the following expected utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, M_t/p_t, h_t),$$

where $\beta \in (0, 1)$ is the discount factor, $M_t$ is holdings of nominal balances, $h_t$ is labour supply, and $p_t$ is the consumer price level. The single-period utility function is specified as:

$$u(\cdot) = \frac{\gamma a_t}{\gamma - 1} \log \left[ c_t^\gamma + b_t^\gamma \left( \frac{M_t}{p_t} \right)^{\frac{2-\gamma}{\gamma}} + \eta \log (1 - h_t),$$

where $\gamma > 0$ and $\eta > 0$ denote the constant elasticity of substitution between consumption and real balances, and the weight on leisure in the utility function, respectively; $a_t$ and $b_t$ are two different preference shocks. We interpret $a_t$ as a taste shock that enters into the Euler equation linking the household’s consumption growth to the real interest rate. The shock $b_t$, however, is interpreted as a shock to money demand. These shocks follow first-order autoregressive processes:

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{at},$$

and

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt},$$

where $\rho_a, \rho_b \in (-1, 1)$ are autoregressive coefficients, $b$ is a constant, and the serially uncorrelated shocks $\varepsilon_{at}$ and $\varepsilon_{bt}$ are normally distributed with zero means and standard deviations $\sigma_a$ and $\sigma_b$, respectively.

The representative household enters period $t$ with $k_t$ units of capital; nominal money balances, $M_{t-1}$; nominal domestic bonds, $B_{t-1}$; and nominal net foreign bonds, $B_{t-1}^*$, denominated in foreign currency. During period $t$, the household may purchase new domestic bonds, $B_t$, and foreign bonds, $B_t^*$, on domestic and international financial markets while
receiving payments from previous-period bond holdings. It also supplies labour and capital to domestic-intermediate firms and receives total factor payments, $R_{kt}k_t + W_t h_t$, where $R_{kt}$ is the nominal rental rate for capital and $W_t$ is the nominal wage rate. Furthermore, it receives a lump-sum nominal transfer, $T_t$, from the monetary authority, and dividend payments from the monopolistically competitive intermediate-goods producers and importers, $D_t^d = \int_0^1 D_t^d(j)\,dj$ and $D_t^f = \int_0^1 D_t^f(j)\,dj$. The household uses some of its funds to purchase, at a nominal price, $p_t$, consumption and investment. Investment, $i_t$, increases the capital stock over time, according to:
\begin{equation}
k_{t+1} + \Psi(k_{t+1}, k_t) = (1 - \delta)k_t + i_t,
\end{equation}
where $\delta \in (0, 1)$ is the constant capital depreciation rate, and $\Psi(\cdot, \cdot)$ is a capital-adjustment cost function specified as $\psi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 k_t$, where $\psi > 0$ is the adjustment cost parameter. With this specification, both total and marginal costs of adjusting capital are zero in the steady-state equilibrium.

The household’s budget constraint is given by:
\begin{align*}
p_t (c_t + i_t) &+ M_t + \frac{B_t}{R_t} + \frac{\kappa_t B_t^*}{\kappa_t R_t^*} \leq R_{kt}k_t + W_t h_t + M_{t-1} \\
& \quad + B_{t-1} + \kappa_t B_{t-1}^* + T_t + D_t^d + D_t^f,
\end{align*}
where $R_t$ and $R_t^*$ denote the gross nominal domestic and world interest rates between $t$ and $t + 1$, respectively; $c_t$ is the nominal exchange rate (the price of the foreign currency in the domestic currency); and $\kappa_t$ is an endogenous country-specific risk premium that reflects departures from uncovered interest rate parity.\(^3\) Thus, in period $t$, the domestic household may purchase foreign bonds, $B_t^*$, for $(\kappa_t R_t^*)^{-1}$ units of foreign output.\(^4\)

The domestic household has access to incomplete international financial markets, but the price it must pay is increasing in the foreign-debt-to-output ratio in the domestic economy.

\(^3\) McCallum and Nelson (1999) and Kollmann (2002) use an exogenous risk-premium term that follows an AR(1) process.

\(^4\) The price of domestic bonds is $1/R_t$ units of domestic output; however, the price of foreign bonds on the international financial market is $1/R_t^*$ units of foreign output. It is assumed that foreigners purchase only the bonds denominated in their own output.
The risk-premium term, $\kappa_t$, is therefore given by:

$$
\kappa_t = \exp \left( -\frac{\varphi c_t \tilde{B}_t^*}{p_{dt}y_t} \right),
$$

where $\varphi$ is a parameter that measures the level of the risk premium, $\tilde{B}_t^*$ is the average stock of aggregate foreign debt, and $p_{dt}$ and $y_t$ are the domestic-output price and real output, respectively.\(^5\) The risk-premium term implies that the equilibrium steady-state is unique and induces stationarity in the model. In models with incomplete asset markets, if $\varphi$ is equal to 0, even when both domestic and world real interest rates are equal to $1/\beta$, there is hysteresis and temporary shocks have permanent effects on the level of macroeconomic variables (Senhadji 1995).\(^6\)

The world gross nominal interest rate, $R^*_t$, is exogenous and evolves according to:

$$
\log( R^*_t ) = (1 - \rho_{R^*}) \log( R^* ) + \rho_{R^*} \log( R^*_{t-1} ) + \varepsilon_{R^*t},
$$

where $\rho_{R^*} \in (-1, 1)$ is the autocorrelation coefficient and the serially uncorrelated shock $\varepsilon_{R^*t}$ is normally distributed with zero mean and standard deviation $\sigma_{R^*}$.

The household chooses \( \{c_t, M_t, h_t, k_{t+1}, B_t, B_t^*\} \) to maximize the expectation of the discounted sum of its utility flows subject to (5) and (6). The first-order conditions are:

$$
\frac{a_t c_t^{-\frac{1}{\gamma}}}{c_t^{-\frac{1}{\gamma}} + \frac{1}{b_t^*} (M_t/p_{t})^{-\frac{1}{\gamma}}} = \lambda_t; \\
\frac{a_t b_t^* (M_t/p_{t})^{-\frac{1}{\gamma}}}{c_t^{-\frac{1}{\gamma}} + \frac{1}{b_t^*} (M_t/p_{t})^{-\frac{1}{\gamma}}} = \lambda_t - \beta E_t \left( \frac{p_t \lambda_{t+1}}{p_{t+1}} \right); \\
\eta \left( \frac{1 - h_t}{\eta} \right) = \lambda_t w_t; \\
\lambda_t (1 - h_t) = \lambda_t w_t;
$$

\(^5\)Senhadji (1995) and Schmitt-Grohé and Uribe (2003) use a functional form for a risk premium that depends only on the aggregate level of foreign debt. An economy is a net debtor if $B^*_t < 0$, and it must pay a risk premium, $\kappa_t$, in addition to $R^*_t$.

\(^6\)In such a case, there is a random walk in equilibrium dynamics, so that one eigenvalue is equal to 1.
\[ \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{R_{k+1}}{p_{t+1}} + 1 - \delta + \psi \left( \frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}} \right) \right] = \psi \left( \frac{k_{t+1}}{k_t} - 1 \right) + 1; \quad (13) \]

\[ \frac{1}{R_t} = \beta E_t \left[ \frac{p_t \lambda_{t+1}}{p_{t+1} \lambda_t} \right]; \quad (14) \]

\[ \frac{1}{\kappa R^*_t} = \beta E_t \left[ \frac{e_{t+1} p_t \lambda_{t+1}}{e_t p_{t+1} \lambda_t} \right]; \quad (15) \]

in addition to the budget constraint, where \( \lambda_t \) is the Lagrangian multiplier of the budget constraint. Equations (14) and (15) together imply:

\[ \frac{R_t}{\kappa_t R^*_t} = \frac{e_{t+1}}{e_t}, \quad (16) \]

the uncovered interest rate parity (UIRP) condition.

### 2.2 Aggregation sector

The aggregator is a perfectly competitive firm that uses differentiated domestic- and imported-intermediate goods to produce domestic- and imported-composite goods. It turns domestic- and imported-composite goods into a final good using a CES production technology.

#### 2.2.1 Domestic- and imported-composite goods

The domestic- and imported-composite goods, \( y_{dt} \) and \( y_{ft} \), are produced using, respectively, a continuum of domestic- and imported-intermediate goods, \( y_{dt}(j) \) and \( y_{ft}(j) \), and the CES aggregate technology:

\[ y_{dt} \leq \left( \int_0^1 y_{dt}(j)^{\frac{\theta - 1}{\theta}} dj \right)^{\frac{\theta}{\theta - 1}}, \quad (17) \]

where \( \theta > 1 \) is the constant elasticity of substitution between different intermediate goods.

Given the domestic output price, \( p_{dt} \), and the price of the domestic-intermediate good, \( p_{dt}(j) \), the competitive firm chooses the quantity of \( y_{dt}(j) \) that maximizes its profit. The maximization problem is

\[ \max_{\{y, a(j)\}} p_{dt} y_{dt} - \int_0^1 p_{dt}(j)y_{dt}(j) dj, \quad (18) \]
subject to (17). The resulting demand function for the domestic-intermediate good is:

$$y_{dt}(j) = \left( \frac{p_{dt}(j)}{p_{dt}} \right)^{-\theta} y_{dt}.$$  

(19)

The domestic output price, which is the producer price index (PPI), satisfies

$$p_{dt} = \left( \int_0^1 p_{dt}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$  

(20)

Similarly, the maximization problem in the import sector implies the following demand function for the imported-intermediate good:

$$y_{ft}(j) = \left( \frac{p_{ft}(j)}{p_{ft}} \right)^{-\theta} y_{ft}.$$  

(21)

In addition, the import price, which is the importer-price index (IPI), satisfies

$$p_{ft} = \left( \int_0^1 p_{ft}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$  

(22)

2.2.2 Final good

The final good, $z_t$, is produced using domestic- and imported-composite goods, and the following aggregate technology:

$$z_t = \left[ (1 - \omega_f)^{\frac{1}{\nu}} y_{dt}^{\frac{\nu - 1}{\nu}} + \omega_f^{\frac{1}{\nu}} y_{ft}^{\frac{\nu - 1}{\nu}} \right]^{\frac{\nu}{\nu - 1}},$$  

(23)

where $\omega_f > 0$ denotes a positive share of imported goods in the production of the final good, and $\nu > 0$ is the elasticity of substitution between domestic and imported goods. The final good is used for home consumption and investment, so that

$$z_t = c_t + i_t.$$  

(24)

Given the price level of the final good, $p_t$, and given $p_{dt}$ and $p_{ft}$, the competitive firm chooses $y_{dt}$ and $y_{ft}$ to maximize its profit. The maximization problem is

$$\max_{\{y_{dt}, y_{ft}\}} p_t z_t - p_{dt} y_{dt} - p_{ft} y_{ft},$$  

(25)
subject to (23). Profit maximization implies the following demand functions for domestic- and imported-composite goods:

\[ y_{dt} = (1 - \omega_f) \left( \frac{p_{dt}}{p_t} \right)^{-\nu} z_t \quad \text{and} \quad y_{ft} = \omega_f \left( \frac{p_{ft}}{p_t} \right)^{-\nu} z_t. \]  

(26)

Thus, as the relative prices of domestic and imported goods rise, the demand for domestic and imported goods decreases. The price elasticity of these demand functions for domestic and imported goods is \( \nu \).

The zero-profit condition implies that the price level of the final good, which is the consumer price index (CPI), is linked to domestic-output and import prices through:

\[ p_t = \left[ (1 - \omega_f)p_{dt}^{-\nu} + \omega_f p_{ft}^{-\nu} \right]^{1/(1-\nu)}. \]  

(27)

### 2.3 Intermediate-goods sector

#### 2.3.1 Domestic-intermediate goods

The market for domestic-intermediate goods is modelled as in the CE models. The domestic producer, \( j \), uses \( k_t(j) \) and \( h_t(j) \) to produce a differentiated domestic-intermediate good, \( y_t(j) \), according to the following constant-returns-to-scale technology:

\[ y_t(j) \leq k_t(j)^{\alpha} [A_t h_t(j)]^{1-\alpha}, \quad \alpha \in (0, 1), \]  

(28)

where \( A_t \) is an exogenous technology shock that is identical for all domestic producers. This shock follows the process

\[ \log A_t = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \]  

(29)

where \( \rho_A \in (-1, 1), A > 0, \) and \( \varepsilon_{At} \) is normally distributed with zero mean and standard deviation \( \sigma_A \). The domestic-intermediate good is then divided between domestic use, \( y_{dt}(j) \), and exports, \( y_{xt}(j) \), so that

\[ y_t(j) = y_{dt}(j) + y_{xt}(j). \]  

(30)
Since it is assumed that the domestic producers cannot price-discriminate, the export price is simply $p_{dt}(j)/e_t$. The foreign demand function for domestic exports is assumed to resemble the domestic demand function (19), and is given by:

$$y_{xt}(j) = \left( \frac{p_{dt}(j)}{p_{dt}} \right)^{-\theta} y_{xt},$$

where $y_{xt}$ is the home country’s aggregate exports. As in McCallum and Nelson (1999) and Kollmann (2001), it is assumed that the total foreign demand for exports is

$$y_{xt} = \left( \frac{p_{dt}}{e_t p_t^*} \right)^{-\tau},$$

where $\tau > 0$ is the price elasticity of the home country’s aggregate exports, and $p_t^*$ is the world price level (denominated in foreign currency). The foreign-price index is assumed to evolve according to:

$$\log\left(\frac{p_t^*}{p_{t-1}^*}\right) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log\left(\frac{p_{t-1}^*}{p_{t-2}^*}\right) + \varepsilon_{\pi^*t},$$

where the world gross inflation rate is $\pi_t^* = \frac{p_t^*}{p_{t-1}^*}$, $\pi^*$ is the world inflation steady-state value, and $\rho_{\pi^*} \in (-1, 1)$ is the autocorrelation coefficient. The serially uncorrelated shock $\varepsilon_{\pi^*t}$ is normally distributed with zero mean and standard deviation $\sigma_{\pi^*}$.

The producer of domestic-intermediate goods sells its output at price $\bar{p}_{dt}(j)$, in monopolistically competitive domestic and foreign markets. Following Calvo (1983), producers of domestic-intermediate goods cannot change their prices unless they receive a random signal. The probability that a given price can be reset in any period is constant and is given by $(1-\phi)$. Therefore, on average the price remains unchanged for $1/(1 - \phi)$ periods.

If a domestic producer, $j$, is allowed to change its price, it chooses $k_t(j)$ and $h_t(j)$ and sets the price, $\bar{p}_{dt}(j)$, that maximizes the expected discounted flow of its profits. The maximization problem is:

$$\max_{\{k_t(j), h_t(j), p_t(j)\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta^\phi)^t \chi_{t+\ell} D_{t+\ell}(j)/p_{t+\ell} \right],$$

where $\chi_{t+\ell} = \left( \frac{\bar{p}_{dt}(j)}{p_{dt}(j)} \right)^{-\theta}$. 

11
subject to (28) and to the following demand functions:

\[ y_{dt+t}(j) = \left( \frac{\bar{p}_{dt}(j)}{p_{dt+t}} \right)^{-\theta} y_{dt+t} \quad \text{and} \quad y_{xt+t}(j) = \left( \frac{\bar{p}_{dt}(j)}{p_{dt+t}} \right)^{-\theta} y_{xt+t}, \]  

(35)

where the profit function is

\[ D^d_{t+t}(j) = \bar{p}_{dt}(j) y_{t+t}(j) - R_{kt+t} b_{t+t}(j) - W_{t+t} h_{t+t}(j). \]  

(36)

The domestic producer’s discount factor is given by the stochastic process \((\beta^l \lambda_{t+l})\), where \(\lambda_{t+l}\) denotes the marginal utility of consumption in period \(t+l\).

The first-order conditions are:

\[
\frac{R_{kt}}{p_t} = \alpha \frac{y_t(j)}{k_t(j)} q_t; \quad (37)
\]

\[
\frac{W_t}{p_t} = (1 - \alpha) \frac{y_t(j)}{h_t(j)} q_t; \quad (38)
\]

\[
\bar{p}_{dt}(j) = \frac{\theta}{\theta - 1} E_t \sum_{t=0}^{\infty} (\beta \phi)^t \lambda_{t+t} y_{at+t}(j) q_{t+t}/p_{t+t}, \quad (39)
\]

where \(q_t\) is the Lagrangian multiplier associated with the production-function constraint. It measures the real marginal cost of the firm in units of final output.

The aggregate domestic price is

\[
p^{1-\theta}_{dt} = \phi p^{1-\theta}_{dt-1} + (1 - \phi) \bar{p}^{1-\theta}_{dt}. \quad (40)
\]

Equations (37) and (38) state that the marginal cost of the inputs should equal their marginal product weighted by the real marginal cost. Equation (39) relates the optimal price to the expected future price of the final good and to the expected future real marginal costs. This condition, together with (40), allows us to derive a New Keynesian Phillips curve.

2.3.2 Imported-intermediate goods

In the home country, a continuum of domestic importers indexed by \(j \in [0, 1]\) import a homogeneous intermediate good produced abroad for the foreign price, \(p^*_t\). Each importer
uses this imported good to produce a different good, \( y_{ft}(j) \), which is sold in a home monopolistically competitive market to produce the imported-composite good, \( y_{ft} \). As in the domestic-intermediate goods sector, importers can change their prices only when they receive a random signal. The constant probability of receiving such a signal is \((1 - \phi)\).

When an importer, \( j \), is allowed to change its price, it sets the price, \( \bar{p}_{ft}(j) \), that maximizes its weighted expected profits, given the price of the imported-composite output, \( p_{ft} \), the nominal exchange rate, \( e_t \), and the foreign price level, \( p_t^* \). The maximization problem is:

\[
\max_{\{p_{ft}(j)\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta \phi)^t \lambda_{t+t}^f D_{t+t}^f(j) / \bar{p}_{ft+t} \right],
\]

subject to

\[
y_{ft+t}(j) = \left( \frac{\bar{p}_{ft}(j)}{p_{ft+t}} \right)^{-\theta} y_{ft+t},
\]

where the profit function is

\[
D_{t+t}^f(j) = (\bar{p}_{ft}(j) - e_{t+t} p_{t+t}^*) y_{ft+t}(j).
\]

In period \( t \), the importer’s nominal marginal cost is \( e_t p_t^* \), so that its real marginal cost is the real exchange rate, \( s_t = e_t p_t^* / p_t \). The importer’s discount factor is also given by the stochastic process \((\beta^t \lambda_{t+t})\). The first-order condition of this optimization problem is:

\[
\bar{p}_{ft}(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{t=0}^{\infty} (\beta \phi)^t \lambda_{t+t} y_{ft+t}(j) e_{t+t} p_{t+t}^*/p_{t+t}}{E_t \sum_{t=0}^{\infty} (\beta \phi)^t \lambda_{t+t} y_{ft+t}(j) / p_{t+t}}.
\]

The aggregate import price is

\[
p_{ft}^{1-\theta} = \phi p_{ft-1}^{1-\theta} + (1 - \phi) \bar{p}_{ft}^{1-\theta}.
\]

Equation (44) governs the optimal setting of the new import price over time. In the absence of price rigidity, \((\phi = 0)\), it implies that the import-price markup, which is the inverse of the real exchange rate, is constant and equal to \( \theta / (\theta - 1) \). This equation, together with (45), allows us to derive a New Keynesian Phillips curve that relates the current and expected import-inflation rates to the real exchange rate.
2.4 Monetary authority

Ireland (2003) proposes a general monetary policy rule that allows the central bank to adjust a linear combination of the nominal interest rate and money growth in response to deviations of output and inflation. Therefore, in this paper it is assumed that the Bank of Canada may manage the short-term nominal interest rate, \( R_t \), in response to deviations of domestic output, \( y_t \), the CPI inflation rate, \( \pi_t = p_t/p_{t-1} \), and money growth, \( \mu_t = M_t/M_{t-1} \), from their steady-state values. Thus, the monetary policy rule evolves according to:

\[
\log(R_t/R) = \omega_y \log(y_t/y) + \omega_\pi \log(\pi_t/\pi) + \omega_\mu \log(\mu_t/\mu) + \log(v_t), \tag{46}
\]

where \( R, y, \pi, \) and \( \mu \) are the steady-state values of \( R_t, y_t, \pi_t, \) and \( \mu_t, \) respectively; \( v_t \) is a monetary policy shock that evolves according to

\[
\log(v_t) = \rho_v \log(v_{t-1}) + \epsilon_{vt}, \tag{47}
\]

where \( \rho_v \in [0, 1] \) is an autoregressive coefficient and the serially uncorrelated shock \( \epsilon_{vt} \) is normally distributed with zero mean and standard deviation \( \sigma_v. \)

With this monetary policy specification, money is endogenous; the monetary authority should adjust the money supply to accommodate money demand. The newly created money is injected into the economy through lump-sum transfers to the households, so that \( T_t = M_t - M_{t-1}. \)

The rule in (46) is a modified Taylor (1993) rule. The original rule is specified for a closed economy and it is in terms of output (PPI) inflation rate. Adapting a rule based on CPI inflation, rather than PPI inflation, is motivated by the facts that (i) the monetary policy impact on the exchange rate may be passed through to the consumer price faster than to the output price, (ii) foreign shocks have faster effects on the consumer price, through the exchange rate, than on the output price, and (iii) the Bank has chosen a CPI measure to implement inflation targeting.\(^8\)

\(^7\)A real exchange rate, \( s_t \), was originally introduced into this monetary policy rule. However, the estimates of its coefficient are too small and statistically insignificant, so it is omitted from the final rule. This is in contrast to Lubik and Schorfheide (2003), who find that the Bank has responded to movements of the first difference of nominal exchange rates.

\(^8\)When we estimate a closed economy, the rule in (46) is specified in terms of PPI inflation.
2.5 Symmetric equilibrium

In a symmetric equilibrium, all domestic-intermediate producers and importers are identical, so that 
\[ y_t = y_t(j), \ y_{at} = y_{at}(j), \ y_{xt} = y_{xt}(j), \ p_{at} = p_{at}(j), \ k_t = k_t(j), \ h_t = h_t(j), \ y_{ft} = y_{ft}(j), \ p_{ft} = p_{ft}(j), \] 
for all \( j \in [0, 1] \) and during each period \( t \geq 0 \). Furthermore, the market-clearing conditions \( M_t = M_{t-1} + T_t, B_t = 0, B^*_t = B^*_t \) for all \( t \geq 0 \).

Let \( r_{kt} = R_{kt}/p_t, w_t = W_t/p_t, m_t = M_t/p_t, \) and \( q_t = \lambda_t/\xi_t \) denote the real rental rate on capital services, real wages, real balances, and the domestic-price markup rate, respectively. Also let \( \pi_{at} = p_{at}/p_{at-1}, \pi_{ft} = p_{ft}/p_{ft-1}, \bar{\pi}_{at} = \bar{p}_{at}/p_t, \bar{\pi}_{ft} = \bar{p}_{ft}/p_t, \bar{\bar{\pi}}_{at} = \bar{\bar{p}}_{at}/p_t, \bar{\bar{\bar{\pi}}} = \bar{\bar{p}}_{ft}/p_t, \) and \( b^*_t = B^*_t/p^*_t \) denote the PPI and IPI inflation rates, the relative prices of domestic and imported goods, and the real foreign bonds, respectively. The equations of the complete non-linear equilibrium system are given in Appendix A.\(^9\)

An approximate solution of the model is obtained by taking a log-linear approximation of each variable, in the symmetric equilibrium system, around its steady-state value. Using Blanchard and Kahn’s (1980) method yields a state-space solution of the form:

\[
\begin{align*}
\hat{s}_{t+1} &= \Phi_1 \hat{s}_t + \Phi_2 \hat{\varepsilon}_{t+1}, \\
\hat{d}_t &= \Phi_3 \hat{s}_t,
\end{align*}
\]

where \( \hat{s}_t \) is a vector of state variables that includes predetermined and exogenous variables; \( \hat{d}_t \) is the vector of control variables; and the vector \( \hat{\varepsilon}_{t+1} \) contains technology, money demand, monetary policy, preference, and world interest and inflation rate shocks. This solution is a restricted vector autoregression (VAR), in that the elements of matrices \( \Phi_1, \Phi_2, \) and \( \Phi_3 \) depend on the structural parameters of the model that describe the household’s preferences, the technologies, and the monetary policy rule. The state-space solution in (48)–(49) is used to estimate and simulate the model.

\(^9\)In Appendix A, the current account, equation (A.22), is obtained by substituting the resource constraint into the budget constraint.
3. **Calibration, Data, and Estimation**

As in previous studies that have estimated closed- and open-economy models, some structural parameters should be set prior to the estimation, since the data used contain only limited information about them.\(^\text{10}\) The discount factor, \(\beta\), is set equal to 0.9897, which implies an annual steady-state real interest rate of 4.16 per cent, matching the average observed in the sample. The steady-state domestic and world gross inflation rates are set equal to 1. The parameter \(\eta\), which denotes the weight put on leisure in the utility function, is set at 1.315, so that the representative household spends roughly one third of its time in market activities. During the estimation procedure, the estimate of \(\psi\), the capital-adjustment cost parameter, converges to non-plausible values (values that are too high), so it is set equal to 15, as in Kollmann (2002). In Ireland (2001), this parameter is set equal to 10.\(^\text{11}\) The parameter in the risk-premium term, \(\varphi\), is set equal to 0.0054, which implies an average risk premium of 98 basis points at an annual rate, consistent with the estimates reported by Clinton (1998) for Canada.\(^\text{12}\)

The share of capital in production, \(\alpha\), and the depreciation rate, \(\delta\), are assigned values of 0.33 and 0.025, respectively; these values are commonly used in the real business cycle models. The parameter that measures monopoly power in the markets for domestic- and imported-intermediate goods, \(\theta\), is set equal to 6, which implies a steady-state markup of price over marginal cost equal to 20 per cent. This value is used in Ireland (2001, 2003) and in Dib (2003). The fraction of imported goods in the final good, \(\omega_f\), is set at 0.28, so that the steady-state ratio of import-to-GDP matches its historical average for Canada during the period 1981–2002.

Using a maximum-likelihood procedure with a Kalman filter, the non-calibrated parameters are estimated for two versions of a SOE model and for a CE model. The first version is a restricted small open-economy (RSOE) model, where the parameters \(\nu\) and \(\tau\), which, respectively, capture the price elasticities of aggregate imports and exports, are assumed to

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\(^\text{10}\) For example, Ireland (2001, 2003), Dib (2002, 2003), and Bergin (2003).

\(^\text{11}\) The value of \(\psi\) is also set at 10 and 20, but the estimated parameters are only marginally affected.

\(^\text{12}\) The value of \(\varphi\) is set at 0.004 and 0.006, but the estimated parameters are only marginally affected.
be equal; i.e., \( \nu = \tau \). The second version is an unrestricted small open-economy (USOE) model, in which \( \nu \) may differ from \( \tau \). The CE model is similar to those with sticky prices estimated by Ireland (2001) for the U.S. economy and by Dib (2002) for the Canadian economy.\(^{13}\) The VAR used to estimate both versions of the SOE model consists of six variables: consumption, the CPI inflation rate, the domestic nominal interest rate, real balances, the world nominal interest rate, and the world inflation rate. To estimate the CE model, however, the VAR consists of only the four domestic variables. Moreover, the PPI inflation rate is used, rather than the CPI inflation rate.

The data used are from Canada and the United States and are quarterly from 1981Q3 to 2002Q4.\(^{14}\) Consumption is measured by real personal spending. The CPI and PPI inflation rates are measured by changes in the CPI and in the GDP price deflator, respectively. The short-term nominal interest rate is measured by the rate on Canadian three-month treasury bills. Real balances are measured by dividing the M2 money stock by the GDP price deflator. The world nominal interest rate is measured by the rate on U.S. three-month treasury bills, while the world inflation rate is measured by changes in the U.S. GDP price deflator. The series for consumption and real balances are expressed in per-capita terms using the Canadian civilian population aged 15 and over. Since the model implies that all variables are stationary, the Canadian series are rendered stationary by regressing the logarithm of each variable on a constant and a time trend. However, the U.S. nominal interest and inflation rates are stationary.

4. Empirical Results

4.1 Estimation results

Table 1 reports the maximum-likelihood estimates of the structural parameters of the RSOE, USOE, and CE models. When the price elasticities of aggregate imports and exports are

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\(^{13}\) Ireland (2001) introduces price rigidity by assuming quadratic adjustment costs.

\(^{14}\) The data used start at 1981Q3 because the Bank effectively abandoned M1 growth targeting by the middle of 1981. In the estimation of the CE model, only the Canadian series are used.
assumed to be equal (the RSOE model), the estimated value of $\nu = \tau$ is about 0.8 and it is statistically significant. This estimated value is consistent with the calibrated value usually used in the literature; for example, Kollmann (2001, 2002) set these parameters at 0.6, while Bergin (2003) assumes that the elasticity of substitution between domestic and imported goods is 1 by choosing a Cobb-Douglas technology to produce the final good.

When the two parameters may be different (the USOE version), the estimated value of $\nu$ is close to 0 and statistically non-significant. The estimated value of $\tau$ is about 1.5 and statistically significant, so exports are relatively highly elastic with respect to domestic price and the real exchange rate. With $\nu$ (the elasticity of substitution between domestic and imported goods) estimated at 0, domestic and imported goods are not substitutes. Thus, this finding suggests that the final good is produced using a Leontief technology, where the imported goods are used in a fixed proportion, $\omega_f$. Smets and Wouters (2002) use such a technology to produce a final good for home consumption and exports. In both estimated versions, the sum of the estimated values of $\nu$ and $\tau$ exceeds one, so the static Marshall-Lerner condition is satisfied. Since the likelihood ratio test does not reject the null hypothesis that $\nu = \tau$, and since the estimates of the remaining parameters are very similar in both versions of the SOE model, only the estimation results of the RSOE model are discussed and they are compared with the results estimated in the CE model.\(^{15}\)

First, the estimated values of $\gamma$, the constant elasticity of substitution between consumption and real balances, are about 0.022 and statistically significant. The parameter $b$, which only with $\gamma$ determines the steady-state ratio of real balances to consumption, is estimated at about 0.69, and it is statistically significant.

The estimated value of the parameter $\phi$, which determines the degree of nominal price rigidity in the domestic- and imported-intermediate goods sectors, is 0.52 in the SOE model, and it is about 0.59 in the CE model. These values imply that, in any given period, only 41 to 48 per cent of the producers and importers of intermediate goods are allowed to change their prices. Thus, on average, the prices of domestic and imported goods remain unchanged

\(^{15}\)The RSOE and USOE models generate very similar impulse responses to different shocks, so only those of the RSOE model are reported.
for 2.10 to 2.44 quarters in both economies. This estimated degree of nominal price rigidity is consistent with previous findings by Dib (2002, 2003). Using a CE framework for the Canadian economy, Dib (2002, 2003) estimates that, on average, domestic prices remain unadjusted for about 2.10 quarters in a standard sticky-price model. In an open-economy model without capital, however, Smets and Wouters (2002) estimate $\phi$ at 0.9. Thus, on average, prices are unchanged for about 10 quarters for the euro area, which is too high. Bergin (2003) also estimates, for Canada, a sizable value for the parameter of the price adjustment costs used in his benchmark model with price and wage rigidities.

Next, the estimates of the monetary policy parameters are reported. The estimated values of $\rho_\pi$ and $\rho_\mu$, the coefficients that measure responses of monetary policy to deviations in inflation and money growth, are positive and statistically significant. The estimated value of $\rho_\pi$ is about 0.81 in all estimated models. The estimated value for $\rho_\mu$ is about 0.20 in the SOE model and 0.55 in the CE model. The null hypothesis that the estimates of $\rho_\mu$ are equal in open and closed economies is easily rejected. Thus, the response of the monetary authority to deviations in inflation is very similar in closed and open economies, but the response to deviations in money growth appears more aggressive when the model is estimated as a closed economy. On the other hand, the estimates of $\rho_y$, the coefficient that measures the response of monetary policy to deviations in output, are close to zero and statistically insignificant in both economies. The estimates of the autoregressive coefficient of monetary policy shocks, $\rho_\nu$, are positive and statistically significant. They are 0.25 and 0.34 in the SOE and CE models, respectively. This result suggests that the Bank marginally tries to smooth the nominal interest rate. The estimates of $\sigma_v$, the standard deviation of the monetary policy shock, are about 0.005 in both models.

The remaining domestic shocks – money demand, technology, and preferences – appear to be moderately persistent in the CE model, with autoregressive coefficients at least equal to 0.82; they are, however, relatively more persistent in the SOE model. The estimates of the standard deviation parameters of domestic shocks indicate that they are highly volatile. They are almost similar in both economies. The autoregressive coefficients in the world nominal interest and inflation rates processes, $\rho_{R^*}$ and $\rho_{\pi^*}$, are moderately persistent, with estimates
of about 0.82 and 0.58, respectively, in both open and closed models. The volatilities of world nominal interest and inflation rates are 0.003 and 0.0025, respectively.

### 4.2 Impulse-response functions

Figures 1, 2, 3, and 4 show impulse responses of some macroeconomic variables to a 1 per cent shock to monetary policy, technology, and the world nominal interest rate using the estimated values of the RSOE and CE models. In Figures 1, 2, and 3, Panels A to E plot and compare the responses of output, consumption, the real interest rate, money growth, and the PPI inflation rate to monetary policy and technology shocks generated by the SOE and CE models. Each response is expressed as the percentage deviation of a variable from its steady-state level.

Figure 1 plots the impulse responses to a 1 per cent positive monetary policy shock in the estimated SOE and CE models. This shock is an exogenous tightening of monetary policy. Overall, in Panels A to E, the responses of different variables to a monetary policy shock are qualitatively similar in the SOE and CE models. Following a tightening of monetary policy, the real interest rate increases in both economies; however, output, consumption, PPI inflation, and money growth fall sharply after the shock, before returning progressively to their steady-state levels.

On the other hand, the increase in the domestic real interest rate appreciates the real exchange rate, which leads to a decrease in exports. The presence of nominal rigidity in the import sector implies a gradual adjustment in the import price, so the relative import price increases in the short term after a monetary policy shock. Imported goods are relatively more expensive than domestic ones. Therefore, domestic agents will substitute more domestically produced goods for imported goods, which means a large decrease in imports and an increase in net exports, unlike the prediction of the theory.\(^1\)

The response of consumption indicates that the intertemporal substitution effect dominates the expenditure-switching effect after the shock; the fall in consumption is sharper in

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\(^1\)When we reduce the degree of import-price rigidity (import prices remain unchanged for only 1.4 quarters), net exports respond negatively to monetary policy shocks in the short and long terms.
the SOE model than in the CE model, in contrast with the prediction of the theory. This result may be explained by the fact that, in the SOE model, the relative import price increases in the short term, which leads to no expenditure switching at all. By reducing the degree of price stickiness in the import sector, however, the relative import price responds negatively to a monetary policy shock, so imported-intermediate goods are cheaper, which leads to a small expenditure-switching effect in the short and long terms. Nevertheless, the expenditure-switching effect marginally exceeds the intertemporal substitution in the long term, so that consumption jumps slightly above its steady-state level in the fifth quarter after the shock.17

To determine how the monetary policy may affect the intertemporal substitution and expenditure-switching effects, it is assumed that the monetary policy rule is identical in the SOE and CE models. Figure 2 shows the impulse responses to a 1 per cent positive monetary policy shock in the SOE and CE models where, in both models, the parameters of the monetary policy rule are set equal to the values estimated in the RSOE version. In this case, the instantaneous responses of output, consumption, and the real interest rate are greater (sharper) in the CE model than in the SOE model. The response of consumption shows that, in the SOE model, the expenditure-switching effect is substantial, and the response of consumption is lower in the SOE model.18 Thus, by assuming an identical monetary policy rule, the SOE and CE models lead to isomorphic effects of monetary policy shocks, though the expenditure-switching effect is relatively important in the short term.

Figure 3 shows the impulse responses of the variables to a 1 per cent positive technology shock.19 The effects of this shock on the domestic variables are qualitatively similar in the

17 To determine whether the degree of the intertemporal substitution of consumption affects these results, SOE and CE models with a CRRA utility function are simulated. With the coefficient of relative risk aversion set at 2, the impulse responses to a positive monetary policy shock show that the effects of this shock are still isomorphic in the small open and closed economies. Nevertheless, consumption responds only marginally; the responses of output, inflation, and money growth are smaller in these models with the larger coefficient of relative risk aversion, but the response of the real interest rate is slightly greater than what it is in the estimated models.

18 In theory, the intertemporal substitution and expenditure-switching effects affect aggregate demand in opposite directions following a change in the real interest rate.

19 In the SOE model, it is assumed that technology shocks decay with an autoregressive coefficient of $\rho_A =$
estimated SOE and CE models. In both, output, consumption, the real interest rate, and money growth increase after the technology shock. The output and consumption responses are highly persistent. Note that the increase in output is much higher in the SOE model than in the CE model, whereas the consumption response is very similar in both economies. The responses of the real interest rate and money growth are moderately, but persistently, negative starting in the second quarter after the shock. These responses reflect the fact that the central bank accommodates technology shocks by moderately decreasing the real interest rate and money growth. As expected, the PPI inflation rate responds negatively to a technology shock in the SOE and CE models.

A positive technology shock also leads to a depreciation of the real exchange rate, because the real interest rate decreases to accommodate technology shocks, and domestic output increases relative to foreign output. After the shock, the IPI inflation rate jumps above its steady-state level, indicating an increase in the relative import price; however, the CPI inflation rate falls significantly before gradually returning to its steady-state level. The decline of the output price and the depreciation of the real exchange rate increase foreign demand for domestic exports, while domestic demand for foreign goods (imports) decreases. Thus, as in Galí and Monacelli (2002) and Smets and Wouters (2002), net exports increase even more due to the expenditure-switching effects. Therefore, the home country is richer following a positive productivity shock that induces domestic households to hold more and more foreign bonds and reduce their foreign debt.20

As in a closed economy, a positive technology shock leads to an easing of monetary conditions as the real interest rate moderately, but persistently, decreases. This easing of monetary policy leads to a depreciation of the real exchange rate, and to a large increase in exports and output.

Figure 4 shows the impulse responses to a 100-basis-point positive shock to the world nominal interest rate. This shock leads to a temporary increase in output and to a decrease in consumption. It also leads to a one-quarter decrease in the real interest rate, PPI inflation rate.

\[0.88, \text{as estimated in the CE model.}\]

20In this economy, the home country is a net debtor, so it reduces its foreign debt stock after a positive home technology shock.
rate, and money growth. The three variables then jump above their steady-state levels before gradually returning.

A positive shock to the world interest rate also substantially depreciates the real exchange rate for at least seven quarters. The real exchange rate depreciates by 3.5 per cent after the shock, which leads to an important increase in exports and to a high decrease in imports. It also leads to an increase in the CPI and IPI inflation rates in the short term. The IPI inflation rate initially jumps by about 4 per cent. Its response becomes negative in the fifth quarter after the shock.

The relative domestic-output price temporarily falls while the relative import price rises. In this case, households gradually raise their net foreign bond holdings in response to a positive shock to the world nominal interest rate due to the improvement in the trade balance.

As Gali and Monacelli (2002) show, there is a positive correlation between domestic and world interest rates. Nevertheless, this correlation does not prevent a sizable depreciation of the real exchange rate, which leads to a large increase in import prices. Therefore, the expenditure-switching effect induces a relatively persistent decrease in consumption.

4.3 Volatility and autocorrelation

This section examines the ability of the estimated SOE model to generate volatility, relative volatility, and autocorrelation functions of some macroeconomic variables, and compares them with those generated by the CE model. Using the values estimated for the structural parameters of the RSOE and CE models, the standard deviations, relative volatilities, and autocorrelation coefficients are calculated for detrended output, CPI inflation, the nominal interest rate, and the real exchange rate.

Table 2 reports the standard deviations, expressed in terms of percentage, and the autocorrelation coefficients as computed in the data and generated by the SOE and CE models. In the data (Panel A), detrended output is relatively volatile, having a standard deviation of 3.38 per cent. The CPI inflation rate and the domestic nominal interest rate are less volatile; their standard deviations are 0.66 and 0.89 per cent, respectively. In contrast, the real exchange rate is highly volatile, having a standard deviation of 9.38; this implies that
its relative volatility with respect to that of output is 2.77. The data also report that these variables are positively and very highly autocorrelated over the short and medium horizons. For instance, at the fifth lag, the calculated autocorrelation coefficients are at least 0.41.

Panel B shows that the SOE model overpredicts the volatility of detrended output, the CPI inflation rate, and the nominal interest rate; their standard deviations are 4.51, 1.47, and 1.39 per cent, respectively. The model, however, generates little volatility; it generates a standard deviation of only 1.57 per cent. The model does generate persistent detrended output and a persistent nominal interest rate (their generated autocorrelation coefficients are very close to those observed in the data), but it is unable to generate a moderately persistent inflation rate.

As Panel A shows, the real exchange rate is highly volatile and persistent in the data; however, the new open-economy models are unable to reproduce this effect. This SOE model is also unable to reproduce the observed persistence of the real exchange rate. Nevertheless, to generate a highly volatile and persistent real exchange rate, as observed in the data, the import price must remain unchanged for at least 3.5 years. This finding is consistent with that of Chari, Kehoe, and McGrattan (2002) that a moderate degree of price stickiness is not sufficient to generate volatile and persistent real exchange rates. In contrast, Bouakez (2002) shows that, by the dependence of the firm’s desired markup on its relative price, the new open-economy models may generate a highly volatile and persistent real exchange rate, though with a moderate price stickiness.

Panel C shows that the CE model underpredicts the volatility of detrended output and the nominal interest rate, but it slightly overpredicts the volatility of inflation. The CE model also underpredicts the autocorrelation coefficients of detrended output and the nominal interest rate. It succeeds, however, in generating an inflation rate that is persistent over the short term.

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21 This result is obtained by simulating the SOE model with the parameter $\phi$ set equal to 0.93 in the import sector.
4.4 Variance decomposition

This section compares the forecast-error variances of some macroeconomic variables for the SOE and CE models. Table 3 decomposes, for the RSOE model, the forecast-error variances for detrended output, the inflation rate, the domestic nominal interest rate, and the real exchange rate that are attributable to each type of home and world shock. Table 4 decomposes the forecast-error variances for detrended output, the PPI inflation rate, and the nominal interest rate as calculated in the CE model.

In Table 3, Panel A shows that technology shocks account for a substantial fraction of output fluctuations in the short term. Technology shocks contribute the most to output variations, at least 78 per cent in the short term. Domestic monetary policy and money-demand shocks explain only 6 and 2 per cent of output volatility, respectively, at the one-quarter-ahead horizon. Table 4, however, reports that, in the CE model, though a technology shock is the most important source of output fluctuations in the short and long terms, monetary policy and money-demand shocks account for significant fractions. These shocks explain 16 and 14 per cent of output fluctuations, respectively, at the one-quarter-ahead horizon.

Most of the closed economy contribution of policy, money demand, and preference shocks that contributed to output fluctuations is transferred to technology and world interest rate shocks in the SOE model. World nominal interest rate shocks contribute significantly to short-term output fluctuations, accounting for about 9 per cent of output fluctuations at the one-quarter-ahead horizon. Nevertheless, world inflation rate shocks have no effect, even in the short term, on domestic output fluctuations. Thus, in both the SOE and CE models, a large amount of the output forecast-error variance is explained by technology shocks.

In Tables 3 and 4, Panel B decomposes the forecast-error variances of the CPI and PPI inflation rates, respectively. In the SOE model, technology shocks are the main source of the CPI inflation fluctuations in the short and long terms, accounting for more than 47 per cent of them in the short term. The monetary policy and world nominal interest rate shocks account for a substantial amount of these fluctuations in the short and medium terms. At the one-quarter-ahead horizon, policy and world interest rate shocks account, respectively, for 30 and 14 per cent of the variations in CPI inflation. Preference shocks contribute further to
long-term variations in CPI inflation. In contrast, money-demand and world inflation shocks explain only an insignificant fraction of the domestic CPI inflation rate, particularly in the short term.

In the CE model, monetary policy shocks contribute most to PPI inflation, even in the medium term. About 51 per cent of the total variance is explained by these shocks at the one-quarter-ahead horizon. Technology and money-demand shocks still explain a substantial fraction of PPI inflation in the short and long terms. Even though preference shocks contribute little to PPI inflation in the short term, the fraction attributed to these shocks increases significantly in the long term. Therefore, overall, the results of the inflation forecast-error variances reported in Table 4 for the CE model are very similar to those found in Dib (2002) while simulating a standard sticky-price model using a CE framework for Canada.

In Tables 3 and 4, Panel C decomposes the forecast-error variances of the domestic nominal interest rate as generated in the SOE and CE models. Technology and preference shocks are the most important factors that determine fluctuations in the domestic interest rate, even in the long term; together, they account for at least 75 per cent at the one-quarter-ahead horizon. In contrast, in Table 4, Panel C shows that preference shocks are the most important source of fluctuations in the domestic interest rate in the short and long terms. At the one-quarter-ahead horizon, these shocks explain more than 88 per cent of the variations in the domestic interest rate. World interest rate shocks also contribute significantly to the variations in the domestic interest rate in the short and long terms. Surprisingly, in both models, monetary policy and world inflation shocks account for only insignificant fractions of the fluctuations in the domestic interest rate at all horizons. Monetary policy is driven by endogenous reactions to other shocks, instead of by exogenous impulses.

In Table 3, Panel D decomposes the real exchange rate forecast-error variance. As expected, world interest rate shocks account for most of the variation in the real exchange rate in both the short and long terms; they explain more than 87 per cent of the forecast-error variance. Monetary policy and world inflation shocks are also important sources of real exchange rate variations in the short and long terms, with each accounting for more than 4 per cent of the variations in the real exchange rate at the one-quarter-ahead horizon. Surprisingly,
technology, money demand, and preference shocks account for very little real exchange rate volatility, whatever the forecast horizon. This result is in contrast with Bergin (2003), who finds that world interest rate shocks account for only a very small fraction of the forecast-error variance of the real exchange rate. Money supply shocks, however, explain the largest fraction of the variations in the real exchange rate in the short and long terms. This result may be explained by the substantial degree of price rigidity estimated in Bergin (2003), and by assuming that monetary policy follows a money supply rule.

5. Conclusion

This paper has developed, for Canada, a structural SOE model with domestic and import price stickiness, à la Calvo (1983). Following Ireland (2003), a monetary policy rule has been specified such that the Bank of Canada is assumed to systematically manage the short-term nominal interest rate in response to deviations in inflation, output, and money growth from their steady-state values. An endogenous risk-premium term that depends on the foreign-debt-to-GDP ratio was introduced to ensure a stationary model. The structural parameters of the SOE model, as well as of a CE model, have been estimated using a maximum-likelihood procedure with a Kalman filter.

Estimation results show that both estimated models lead to similar estimates for the Canadian economy. The degree of price stickiness is almost the same in both economies. The estimates for the coefficients of the monetary policy rule are quite similar. Simulation results also show that, overall, the effects of shocks to domestic monetary policy in a small open economy are qualitatively very similar to those derived for a closed economy. Nevertheless, the expenditure-switching effect, due to changes in the real exchange rate and the terms of trade, leads to a small impact on consumption and output. The estimated SOE model is also able to generate high volatility and persistent output, inflation, and the nominal interest rates. The volatility and persistence it generates for the real exchange rate are very small. To generate a highly volatile and persistent real exchange rate, like those observed in the data, the import prices must remain unchanged for at least 3.5 years.
The estimation and simulation results show that monetary policy shocks lead to similar effects on macroeconomic variables in both SOE and CE models. Thus, this main result supports the argument of Clarida, Galí, and Gertler (2001) that the optimal policy problem for a small open economy is isomorphic to that for a closed economy. Therefore, though Canada is a small open economy, using a closed-economy framework is useful to estimate and simulate such a general-equilibrium model to address issues that do not require an open-economy framework. In future work, we will extend the SOE model to include nominal-wage rigidity, price discrimination in home and foreign markets, and different degrees of domestic and import price-stickiness. We will also extend the SOE model to include more domestic and foreign shocks to estimate the model with other relevant series.
References


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Table 1: Maximum-likelihood estimates and standard errors: 1981Q3 to 2002Q4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>RSOE model</th>
<th>USOE model</th>
<th>CE model</th>
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<td></td>
<td>Est.</td>
<td>Std. er.</td>
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Note: $LL$ is the maximum log-likelihood value.
Table 2: Standard deviations and autocorrelation coefficients

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<th>(\sigma_{\hat{x}}/\sigma_y)</th>
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Note: \(\sigma_{\hat{x}}\) is the standard deviation of the variable \(\hat{x}_t\), where \(\hat{x}_t = \hat{y}_t, \hat{n}_t, \hat{R}_t\), or \(\hat{s}_t\).
**Table 3:** Forecast-error variance decomposition: The SOE model

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**Table 4:** Forecast-error variance decomposition: The CE model

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Figure 1: The effects of monetary policy shocks in the estimated SOE and CE models
Figure 2: The effects of monetary policy shocks in the estimated SOE and CE models: Identical monetary policy rule
Figure 3: The effects of technology shocks in the estimated SOE and CE models
Figure 4: The effects of world interest rate shocks in the estimated SOE model
Appendix A: The Non-Linear Equilibrium System

\[
\frac{a_t c_t^{-\frac{1}{\eta}}}{c_t^{-\frac{1}{\eta}} + b_t m_t^{-\frac{1}{\eta}}} = \lambda_t^\eta; \quad (A.1)
\]

\[
\frac{a_t b_t^\frac{1}{\eta} m_t^{-\frac{1}{\eta}}}{c_t^{-\frac{1}{\eta}} + b_t^\frac{1}{\eta} m_t^{-\frac{1}{\eta}}} = \lambda_t - \beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right); \quad (A.2)
\]

\[
\frac{\eta}{1 - h_t} = \lambda_t w_t; \quad (A.3)
\]

\[
\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( r_{kt+1} + 1 - \delta + \psi \left( \frac{k_{t+2}}{k_{t+1}} \right) - 1 \right) \right] = \psi \left( \frac{k_{t+1}}{k_t} - 1 \right) + 1; \quad (A.4)
\]

\[
\frac{\lambda_t}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right]; \quad (A.5)
\]

\[
\frac{R_t}{\kappa_t R_t^*} = E_t \left[ \frac{s_{t+1 \pi_{t+1}}}{s_{\pi_{t+1}}^*} \right]; \quad (A.6)
\]

\[
\kappa_t = \exp \left( -\frac{\varphi s^t b_t^t}{\rho t \pi_t^*} \right); \quad (A.7)
\]

\[
y_t = k_t^\alpha (A_t h_t)^{1-\alpha}; \quad (A.8)
\]

\[
r_{kt} = \frac{\alpha y_t}{k_t} q_t; \quad (A.9)
\]

\[
w_t = \frac{(1 - \alpha) y_t}{h_t} q_t; \quad (A.10)
\]

\[
\mu_t = \frac{m_t \pi_t}{m_t - 1}; \quad (A.11)
\]

\[
\tilde{p}_{dt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+ly_{dt+l}} q_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+ly_{dt+l}} \prod_{i=0}^{l} \pi_t^{-1}}; \quad (A.12)
\]

\[
\phi(1/\pi_{dt})^{1-\theta} + (1 - \phi) \tilde{p}_{dt}^{1-\theta} = 1; \quad (A.13)
\]

\[
\tilde{p}_{jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+j_{yt+l} t} q_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta \phi)^l \lambda_{t+j_{yt+l} t} \prod_{i=0}^{l} \pi_t^{-1}}; \quad (A.14)
\]

\[
\phi(1/\pi_{jt})^{1-\theta} + (1 - \phi) \tilde{p}_{jt}^{1-\theta} = 1; \quad (A.15)
\]

\[
z_t = c_t + k_{t+1} + \psi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 k_t - (1 - \delta) k_t; \quad (A.16)
\]

\[
\log (R_t/R) = \varphi_1 \log (\pi_t / \pi) + \varphi_2 \log (y_t/y) + \varphi_3 \log (\mu_t / \mu) + \log (\nu_t); \quad (A.17)
\]
\begin{align}
y_{it} &= (1 - \omega_J)\bar{p}_{at}^{1-\nu}z_t; & (A.18) \\
y_{ft} &= \omega_J\bar{p}_{ft}^{1-\nu}z_t; & (A.19) \\
y_{xt} &= \left(\frac{\bar{p}_{dt}}{s_t}\right)^{-\nu}; & (A.20) \\
y_t &= y_{at} + y_{xt}; & (A.21) \\
\frac{b_t^*}{\kappa_t R_t^*} - \frac{b_{t-1}^*}{\pi_t^*} = \frac{\bar{p}_{dt}y_{xt}}{s_t} - y_{ft}; & (A.22) \\
(1 - \omega_J)\bar{p}_{at}^{1-\nu} + \omega_J\bar{p}_{ft}^{1-\nu} = 1; & (A.23) \\
\pi_{at} &= \frac{\pi_t \bar{p}_{at}}{\bar{p}_{at-1}}; & (A.24) \\
\pi_{ft} &= \frac{\pi_t \bar{p}_{ft}}{\bar{p}_{ft-1}}; & (A.25) \\
\log(a_t) &= \rho_a \log(a_{t-1}) + \varepsilon_{at}; & (A.26) \\
\log(b_t) &= (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; & (A.27) \\
\log(A_t) &= (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; & (A.28) \\
\log(v_t) &= \rho_v \log(v_{t-1}) + \varepsilon_{vt}; & (A.29) \\
\log(R_t^*) &= (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^*t}; & (A.30) \\
\log(\pi_t^*) &= (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi_{t-1}^*) + \varepsilon_{\pi^*t}. & (A.31)
\end{align}