What Are the Determinants of Long-Term Nominal Interest Rates?

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Abstract

The determinants of long-term nominal interest rates have not yet been fully explained by either economic theory or empirical studies. Since long-term nominal interest rates are the sum of long-term real interest rates and inflation expectations, any macroeconomic factor that impacts expected inflation, real rates or both should affect long-term nominal interest rates. The objective of this paper is to examine how the dynamics of nominal bond yields is related to macroeconomic fundamentals. We consider a structural VECM where identification is achieved by imposing long-run restrictions. A technical innovation of the paper is the identification of structural stochastic trends in a VECM including exogenous variables, which enables us to address the special features of a small-open economy like Canada. We then assess the impact of various shocks - monetary, fiscal and supply shocks - on nominal bond yields. Three main results emerge from our analysis. First, an unexpected permanent fiscal deterioration results in large increases in long-term nominal interest rates. Second, in the long-run, supply shocks have no significant impact on long-term nominal rates. Finally, a permanent shock to inflation results in higher nominal long rates.

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Keywords: long-term nominal interest rates, structural cointegrated model

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1. Introduction

Although many theoretical and empirical studies have been devoted to understanding the determinants of long-term nominal interest rates, controversies still exist regarding the role of economic fundamentals in interest rate dynamics. The long-term nominal interest rate is the sum of the long-term real interest rate and inflation expectations, thus any factor that impacts expected inflation, the real rate or both should affect long-term nominal interest rates. While economic theory suggests that real long-term interest rates are influenced by potential GDP, households’ time preference and the rate of return on investment, inflation expectations are strongly influenced by monetary policy, which depends itself on the various macroeconomic variables that enter the central bank’s reaction function. Macroeconomic shocks should, therefore, have a role to play in explaining long-term nominal interest rates. Existing literature indicates that monetary policy is widely viewed as an important determinant of long-term nominal interest rates, while the impact of fiscal policy and supply shocks on long-term yields remains an open issue with no clear-cut conclusion.

The objective of this paper is to examine how the dynamics of nominal bond yields is related to macroeconomic fundamentals. To that end, we specify a structural vector-error-correction model following the methodology of King, Plosser, Stock, and Watson (1991), where identification is achieved by imposing long-run restrictions. By using long-run restrictions, this methodology is similar to the one proposed by Blanchard and Quah (1989) except that it incorporates the information contained in the cointegrating vector. We first formally test for the presence of cointegration. Our results effectively support the existence of an equilibrium relationship between interest rates and the fundamentals we consider. We next use this relationship to specify a structural VAR in error correction form.

A technical innovation of the paper is the identification of structural stochastic trends in a VECM including exogenous variables, which addresses the special features of a small-open economy. This methodology allows us to assess the importance of various disturbances—defined in terms of monetary, fiscal and supply shocks—as sources of movements in nominal bond yields. Moreover, it provides a convenient way to assess the level of nominal interest rates consistent with the funda-
mentals. The focus on the long-run impact has the advantage of filtering out temporary responses of public policies to business cycle movements. As a result, it is easier to make the distinction between genuine (fiscal and monetary) policy shocks and systematic (business cycle-related) reactions to stabilize economic activity in the short run. The methodology is applied in the Canadian context over the 1962-2003 sample.

Three main results emerge from our empirical analysis. First, budget deficits have a sizeable effect on interest rates. More specifically, an unexpected permanent fiscal deterioration - defined as a one percentage point increase in the primary deficit-to-GDP ratio - results in a 250 basis points increase in long term nominal interest rates. This impact - higher than what is generally found in existing studies - can be explained by the methodology used here to assess the impact of fiscal policy on interest rates. More precisely, within a VAR framework, the results provide an estimate of the impact of unexpected movements—basically the structural shocks—and not an estimate of the systematic component of the variables in the model. Furthermore, the structural shocks are defined in term of permanent shifts. Consequently, such long-lasting movements in fundamentals have a stronger impact on interest rates than temporary movements.

Second, we provide additional evidence regarding the importance of monetary shocks in the dynamics of nominal variables, thus confirming the impact of monetary policy on the inflationary component of nominal interest rates. A one per cent permanent unexpected rise in inflation increases the long-term nominal interest by around 0.6 percent in the long-run.

Finally, in the long-run, we find that supply shocks have no significant impact on long-term nominal interest rates.

The remainder of the paper is organized as follows. Section 2 reviews existing literature regarding the potential determinants of long-term nominal interest rates. Section 3 explains the methodology we use to achieve identification in a structural VECM in the case of a small-open economy. Section 4 presents the main results. Section 5 concludes.
2. Literature survey

The determinants of long-term nominal interest rates remain a debated issue, both theoretically and empirically.

According to the widely accepted Fisher relationship, the long-term nominal interest rate is equal to the sum of the long-term real interest rate and inflation expectations:

\[ i_{nt} = r r_{nt} + \dot{p}_t \tag{2.1} \]

where \( i_{nt} \) is the long-term nominal interest rate, \( r r_{nt} \) the real long-term interest rate and \( \dot{p}_t \) the long-run expected inflation.

Any macroeconomic variable able to impact expected inflation, the real rate or both should thus affect long-term nominal interest rate. Economic theory effectively suggests that real interest rates are influenced by several macroeconomic factors - potential GDP, the rate of return on investment, households’ time preference and investors’ behavior towards risk (see Orr, Edey and Kennedy 1995; Evans and Marshall 2001; Laubach and Williams 2003 among others). Fiscal policy is another potential factor influencing real interest rates, but the relationship between fiscal policy and interest rates remains a vigorously debated issue with no clear-cut conclusion to date. On the other hand, inflation expectations are mainly influenced by monetary policy, which depends itself on the various macroeconomic variables that enter the central bank’s reaction function. Consequently, it seems highly possible that macroeconomic factors have a key role to play in explaining long-term nominal interest rates.

The existing literature on the determinants of long-term nominal interest rates generally considers individually one of those potential factors.\(^1\) The purpose of this section is to survey this literature. After a brief presentation of the two “competing/complementary” theories of the long-term interest

\(^1\) The main exception is Evans and Marshall (2001), who consider a wide set of macroeconomic shocks - including a monetary policy shock, a fiscal policy shock, supply and demand shocks- and study the impact of those shocks on the yield curve.
rate, we consider the impact of macroeconomic shocks on nominal interest rates before focusing on the impact of monetary and fiscal policy on interest rates.

2.1 Two “competing/complementary” theories of the determinants of long-term interest rates

At the theoretical level, two theories coexist to explain long-term interest rates: the first one is based on the expectation hypothesis, while the second one relies on the loanable funds model.

According to the expectation hypothesis, the long-term interest rate can approximate long-run expectations about the future value of short-term rates (plus maturity premiums):

\[
(1 + i_n)^n = (1 + i_1)(1 + i_{e,1})(1 + i_{e,2})(1 + i_{e,3})\ldots(1 + i_{e,n-1})(1 + \text{premium}_n)
\] (2.2)

where \(i_n\) is the long-term interest rate (n years), \(i_1\) the short-term interest rate (one year), \(i_{e,k}\) the short-term interest rate expected to prevail k years ahead (k=1 to n-1) and \(\text{premium}_n\) is the maturity premium (increases with the maturity).

This approach gives current and expected future monetary policy an important role in explaining long-term interest rates since it is widely accepted that monetary policy affects short-term market interest rates (see 2.3).

In the loanable funds model, the long-term real interest rate is the equilibrium price resulting from the demand and the supply of loanable funds in the economy. Therefore, the long-term real interest rate can be influenced by the various factors affecting the demand and supply of funds in the economy. The supply of loanable funds comes from domestic saving - private and public - and, because of the integration of international capital markets, from foreign saving, while the borrowing needs come from the private and the public sectors. If one of those elements is modified, everything else unchanged, the long-term interest rate should be affected (see Box 1 for the case of a fiscal deterioration as an illustration).
2.2 Macroeconomic shocks and long-term interest rates

As explained before, both components of long-term nominal interest rates are potentially affected by macroeconomic variables. Existing literature regarding the impact of macroeconomic shocks on long-term interest rates is, however, rather limited. The main contributions are Evans and Marshall (2001), Ang and Piazzesi (2003) and Wu (2003). With similar US data, however, their conclusions are not the same.

Using a structural VAR approach with different identification strategies, Evans and Marshall (2001) consider the impact of both demand (preference) and supply (technology) shocks on the US yield curve, on the 1959:1-2000:12 period. Whatever the identification strategy, they show that aggregate demand shocks induce the largest, most significant and most persistent responses in nominal yields, because demand shocks move the real interest rates and inflation in the same direction. Regarding the impact of aggregate supply (technology) shocks, on the contrary, they do not obtain robust conclusions. In the context of a structural VAR based on Gali’s identification strategy, the responses of nominal yields to a supply shock are not statistically significant, reflecting the opposite moves in real interest rate and inflation following a supply shock. In the case of a structural VAR identified from model-based shocks, the impact of the technology shock on nominal yields is sensitive to the features of the VAR system (over- versus exactly-identified) and to the ordering of shocks (in exactly-identified systems): technology shocks induce a significant response of the nominal yield level in over-identified systems and in exactly-identified systems where technology shock enters the system after the demand shock, but have no impact if the demand shock precedes the technology shock. The response of nominal yields following a supply shock is therefore particularly sensitive to the identification strategy, which conducts Evans and Marshall (2001) to conclude that this remains an open question.

Ang and Piazzesi (2003) use a Vector Autoregression model where identifying restrictions are

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1. They first use the approach proposed by Gali (1999) where identification is achieved with strong *a priori* restrictions on the covariance structure of the VAR innovations. They next identify a structural VAR by using model-based shock measures. In that case, the identifying restrictions are closely tied to specific economic theories and few prior restrictions are placed on the covariance structure of the VAR innovations.
2. Their results regarding monetary and fiscal policy shocks are presented in 2.3 and 2.4 respectively.
based on the absence of arbitrage to investigate how macroeconomic variables (inflation and real activity) as well as unobservable factors affect the dynamics of the US yield curve with data covering the 1952:6-2000:12 period. Variance decompositions show that macroeconomic factors explain movements at the short and middle ends of the yield curve while unobservable factors still account for most of the movement at the long end of the yield curve. Therefore, Ang and Piazzesi’s conclusions do not support the idea that macroeconomic variables affect long-term nominal interest rates.

In the context of a structural VAR framework where shocks are identified using a recursive strategy with US data covering the 1967:1-1998:12 period, Wu (2003) shows that a positive shock to real output raises all the interest rates with a similar magnitude along the yield curve. Moreover, this effect on the level of the yield curve is more persistent than the effect created by a monetary policy shock. Wu’s results support thus the idea that a supply shock impacts interest rates.

To date, there is hence no firm conclusion in the literature regarding the effect of macroeconomic shocks on long-term nominal interest rates.¹

2.3 Monetary policy and long-term nominal interest rate

Both the Fisher relation and the expectations hypothesis give monetary policy an a priori role in determining long-term nominal interest rates.

Since inflation is ultimately a monetary phenomenon (Bullard 1999), long-run inflation expectations are largely set by monetary policy, making thus monetary policy a relevant candidate as a determinant of nominal interest rates. Several empirical studies (conducted with US data) effectively support the view that long-term nominal interest rates are affected by monetary policy through its impact on inflation.²

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¹ Moreover, the longest-term interest rate used in those studies is the 60-month zero coupon bond yields, which can be viewed as a medium-term rather than long-term interest rate. In our empirical study, we use the 10-year bond yield as the long-term interest rate.

² Figure 1 illustrates the rather similar movements in nominal bond yields and inflation over time in Canada.
Using monthly data on the 1952:1-1987:2 period, Campbell and Ammer (1993) show that bond returns are largely driven by news about future inflation, while real rates have little impact. They find however a small difference in the variance decomposition of bond returns according to the sample period: while the variation in bond returns is essentially explained by news about future inflation over the 1952-1979 period, the news about future excess bond returns also contributes to the overall variance of bond returns in sample periods that include the 1980s. Using cointegration and error-correction methodology in a multivariate framework, Mehra (1996) finds a long-run equilibrium relationship between the US bond rate and the inflation rate that can be interpreted as a Fisher relation in which the (trend) rate of inflation determines the bond rate. The long-run effect of monetary policy on bond yields occurs therefore primarily through the inflation channel.\(^1\) Finally, on the basis of the Lucas’s generalization of the Fisherian theory,\(^2\) Ireland (1996) shows that movements on the long-term bond rate primarily reflect changes in long-run inflationary expectations.

Moreover, as noted before, the expectation hypothesis gives current and expected future monetary policy an important role in explaining long-term interest rates.

Empirical studies generally find a weak relationship between monetary actions and long-term interest rates (see Roley and Sellon 1995 for a detailed survey of those empirical studies in the US case), questioning hence the ability of the monetary authority to influence longer-term interest rates and, eventually, aggregate demand. In the context of structural VAR models, Evans and Marshall (2001) show that monetary policy shocks have a significant impact on the slope of the yield curve, but no effect on the level of the yield curve. Wu (2003), with a similar approach, confirms that monetary policy shocks have a large and significant but short-lived effect on short-term interest rates with a dissipating effect on longer-term interest rates.

This weak impact of monetary policy on long-term nominal interest rates can be explained by the

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1. In the short-run, however, Mehra (1996) finds that monetary policy also affects the real rate component. This is, nevertheless, out of the scope of our study which focuses on the long-run.
2. Lucas (1978) generalizes the Fisher relation by identifying a risk premium as the third determinant of the nominal bond yields. This risk premium compensates investors for holding dollar-denominated bonds in a context characterized by inflation uncertainty.
fact that previous studies only consider current monetary policy, while the expectations theory relates long-term interest rates not only to the current short rate but also to market expectations of future short-term rates. Any change in the view of market participants about future monetary policy can consequently affect the long-term interest rate. By explicitly including market expectations of future monetary actions, Roley and Sellon (1995) find a larger response of long-term interest rates to monetary policy than traditionally: they show that the magnitude of the response of long-term rates to monetary actions depends on the expected persistence of those actions. A change in the current short-term interest rate influences hence longer rates only if market participants view this change as relatively permanent or as the first of a series of actions. The effect of monetary policy on long-term nominal interest rates is thus linked with the persistence of this policy.

In summary, existing literature shows that, in the long run, monetary policy is able to impact on long-term nominal interest rates through its inflation component, that is in affecting inflation expectations.

2.4 Fiscal policy and long-term interest rates

The relationship between fiscal policy and long-term interest rates is a vigorously debated issue, both theoretically and empirically. Moreover, it is a politically sensitive issue for which there is no clear conclusion.  

There are both elements that indicate that fiscal policy should not influence long-term interest rates, and others that suggest an impact of the fiscal position on long-term interest rates.

According to the Ricardian equivalence (Barro 1974), economic agents understand that any increase in current fiscal deficits will conduct to tax raises in the future. To smooth their consumption over time, economic agents therefore increase their present saving in face of the higher fiscal deficits. This parallel increase in both private saving and public borrowing needs results thus in unchanged long-term interest rates. As a result, fiscal policy does not influence interest rates.

Moreover, since international asset markets are increasingly integrated, the relationship between domestic saving and borrowing needs has necessarily weakened, reducing therefore the potential impact of domestic fiscal policy on interest rates.

Nevertheless, the evolution of fiscal positions, savings rate and long-term interest rates over the 1980s and the 1990s in the main industrialized countries has questioned the relevance of the Ricardian equivalence.

The following box illustrates how a deteriorating fiscal position can impact on long-term real interest rate through its impact on the supply of loanable funds (in face of unchanged borrowing needs
Situation [1] is relatively implausible because empirical studies have shown that only 20 to 50 percent of a decrease in public saving is offset by a rise in private saving. Situation [A] is also implausible because empirical evidence suggests that changes in net foreign investment flows account only for 25 to 40 percent of changes in national saving (Feldstein and Horioka 1980; Obstfeld and Rogoff 2000). Consequently, situation [B] is the most realistic one given existing empirical evidence about the behavior of private and foreign saving in face of a deteriorating fiscal position:

1. See Gale and Orzag (2003), pages 6-7 for a detailed survey of those empirical studies.
fiscal policy should thus impact long-term interest rates according to the loanable funds approach.¹

To date, there is no definitive conclusion about which of the previous arguments is correct, the Ricardian view or the approach based on the loanable funds model. However, several large-scale macroeconometric models have detected an economically significant link between changes in fiscal position and long-term interest rates: a one per cent increase in the fiscal deficit-to-GDP ratio would raise long-term interest rates by about 100 basis points after 10 years (see Gale and Orzag 2003, Table 1, page 18), providing thus an additional argument in favor of a link between fiscal policy and long-term interest rates.²

The debate is also far from being close at the empirical level because existing empirical studies of the relationship between fiscal policy and long-term interest rates have produced mixed results.³

In the Canadian case, existing empirical studies give mixed results too. Siklos (1988), with spectral analysis and time series (with annual and quarterly data), finds no evidence to support the view that fiscal deficits influence interest rates (real and nominal). Nunes-Correia and Stemitsiotis (1993) find that, in Canada, a one percentage point increase in the deficit ratio creates a 53 basis points increase in the long-term interest rate. Furthermore, they show that the average fiscal deficit ratio has induced a 236 basis points increase in the long-term interest rate over the 1980-90 period, concluding therefore that fiscal deficits have been an important determinant of long-term interest rates in the 1970s and 1980s. Finally, using a VECM approach on the 1972-1994 period, Fillion (1996) finds a strong cointegration relationship between real long-term interest rates in Canada and the United-States and the Canadian public debt. He next shows that a simulated public debt shock (one percentage point increase in the public debt ratio) induces a 3.1 basis points increase in long-term real interest rates and concludes that the public debt increase in Canada from 1990 has induced a 85 to 135 basis point increase in real interest rates.

¹. Moreover, fiscal policy can impact on interest rates because an increasing public debt creates an eviction effect on capital, rising thus the capital returns and, consequently, the returns on other assets, including Government bonds.
². Moreover, if there is a risk of monetization of increasing fiscal deficits, inflation expectations could be affected upwards following a fiscal deterioration. In that case, fiscal policy would influence nominal interest rates not only through its impact on real interest rates as described before, but also through its impact on (expected) inflation.
³. See Brook (2003) and Gale and Orzag (2003) for very detailed surveys of those empirical studies.
It is more and more widely accepted that the temporal dimension of the relationship between fiscal policy and interest rates must be taken into account in studying this relationship (Feldstein 1986; Brook 2003; Gale and Orzag 2003). Because of the forward-looking nature of financial markets, long-term interest rates respond to expectations of future fiscal policy, rather than to the current policy stance. Studies using projected fiscal deficits effectively find a positive and significant impact of expected fiscal deficits on expected future interest rates.¹

Using the Congressional Budget Office (CBO) budget balance projections, Canzoneri, Cumby and Diba (2002) find that a one per cent of current GDP increase in the projected future deficits raises the spread between long and short term interest rates by 53 to 60 basis points for the five-year projections and by 41 to 45 basis points for the ten-year projections.

Laubach (2003) measures the impact of both the CBO projections and the Office of Management and Business (OMB) projections on the real five-years-ahead ten-year treasury yield, while controlling for other variables viewed as influencing the long-term interest rate (potential GDP growth and equity premium). He finds that a one per cent increase in the projected deficit-to-GDP ratio induces a 28 to 40 basis points rise in the long-term interest rate in the future and that a one per cent increase in the projected debt-to-GDP ratio raises the interest rate by 5.2 basis points, which is consistent with economic theory (neoclassical model of growth).

Finally, following Feldstein (1986), it is more and more widely accepted that the potential impact of fiscal policy on long-term interest rates depends on the “nature” of the fiscal situation: the fiscal position can affect long-term interest rates only if it is viewed as permanent or structural, while a temporary fiscal deterioration - aiming to fight an economic slowdown - has no impact on interest rates.

¹ These studies are based on US data only because of the data availability of fiscal projections.
3. Statistical Framework

As mentioned in the introduction, the objective of this paper is to examine how the dynamics of Canadian nominal bond yields is related to several macroeconomic factors. In most of macroeconomic models, including the recent dynamic general equilibrium models, cycles are usually driven by some combination of monetary, fiscal, and technological innovations. We can therefore reasonably assume that nominal interest rates respond to the news coming from these macroeconomic impulses. In that section, we propose an econometric approach that allows us to assess the importance of various disturbances as sources of movements in nominal bond yields.

We specify the following vector-error-correction model (VECM):

\[
\begin{bmatrix}
\Delta \text{INF}_t \\
\Delta \text{GDP}_t \\
\Delta \text{DEF}_t \\
\Delta r_t
\end{bmatrix}
= \sum_{i=1}^{p-1} \Gamma_i
\begin{bmatrix}
\Delta \text{pcom}_{t-i} \\
\Delta \text{INF}_{t-i} \\
\Delta \text{GDP}_{t-i} \\
\Delta \text{DEF}_{t-i} \\
\Delta r_{t-i}
\end{bmatrix}
+ \alpha \beta'
\begin{bmatrix}
\text{pcom}_{t-i} \\
\text{INF}_{t-i} \\
\text{GDP}_{t-i} \\
\text{DEF}_{t-i} \\
r_{t-i}
\end{bmatrix}
+ \mu + \epsilon_t. \tag{1}
\]

The endogenous variables are the following quarterly Canadian variables: the Consumer Price Index (CPI) year-over-year inflation rate, the real gross domestic product (GDP), the government primary balance - that is the government net lending excluding the interest debt service - expressed as a share of GDP (DEF), and the 10-year government bond rate (r). Given that the set of lagged variables is assumed to be a good proxy for the information set available to economic agents, we also include commodity prices (pcom) as an exogenous variable since Canada is a small open economy which exports mainly primary products and is, thereby, highly sensitive to commodity price developments. We estimate this VECM specification over the 1962-2003 period using a 10 lags structure, \(^1\) which is consistent with the usual information criteria (Hannan-Quinn and Schwartz) and large enough to remove the residual autocorrelation. Unit root tests suggest that all the variables are integrated of order one; the variables are therefore specified in first difference in our model.\(^2\)

\(^1\) Our results are qualitatively robust to specifications with 9 and 11 lags.
\(^2\) There exists some evidence that inflation may have become stationary since the adoption of an inflation targeting regime in 1991 [St-Amant and Tessier (2000)]. However, over a longer period - as the one we use here - formal unit-root tests tend to support the nonstationarity hypothesis. Cogley and Sargent (2001) argue that there has been a downward shift in the degree of persistence in inflation in the US, while others (see Stock, 2001) consider that the statistical evidence in favour of such a break is weak.
The results obtained from cointegration tests corrected for the presence of one exogenous variable, as proposed by Pesaran, Shin and Smith (2000), are presented in Table 1. Both the L-max and Trace tests indicate the presence of 1 cointegration vector.

Table 1: Cointegration Tests

<table>
<thead>
<tr>
<th>L-max</th>
<th>Trace</th>
<th>H0:r=</th>
<th>L-max (.10)</th>
<th>Trace (.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>39.46</td>
<td>57.86</td>
<td>0</td>
<td>28.2</td>
<td>54.84</td>
</tr>
<tr>
<td>14.32</td>
<td>18.39</td>
<td>1</td>
<td>22.1</td>
<td>35.8</td>
</tr>
<tr>
<td>3.98</td>
<td>4.08</td>
<td>2</td>
<td>15.9</td>
<td>20.7</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>3</td>
<td>9.5</td>
<td>9.5</td>
</tr>
</tbody>
</table>

1. The critical values corrected for the presence of one exogenous variable are taken from Table T.3 in Pesaran et al. (2000).

Table 2: Testing restrictions on the cointegrating vector

<table>
<thead>
<tr>
<th>INF</th>
<th>GDP</th>
<th>DEF</th>
<th>LR</th>
<th>pcom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.36</td>
<td>-0.077</td>
<td>-2.408</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(.37)</td>
<td>(.026)</td>
<td>(.38)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The LR test, $\chi^2(1) = 2.3$, p-value = 0.13

a. Standard errors are given within parentheses.

The coefficients of the cointegrating relationship cannot be interpreted as elasticities even if the variables are in logarithm form, because a shock to one variable usually induces a shock to all the variables in the long run. Hence, the coefficients do not generally allow for a *ceteris paribus* interpretation [see Lütkepohl (1994)]. More formally, Wickens (1996) shows that reduced-form cointegration vectors should not be interpreted without further structural assumptions.1

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1. Interpreting those coefficients on a *ceteris paribus* basis would tell that a permanent increase in inflation is associated with a permanent decrease of nominal interest rates. We will illustrate below how such an interpretation is at odd with the impact of structural shocks.
King et al. (1991) [KPSW hereafter] propose an identification methodology based on long-run restrictions that allow for a structural interpretation of a cointegrated VAR. In order to address the special features of a small-open economy like Canada, this paper proposes a technical innovation that allows the identification of structural stochastic trends in a VECM including exogenous variables.\footnote{In Appendix A, we show that the KPSW methodology can be generalized in the context of a VECM with exogenous variables provided the exogenous variables are not cointegrated with the endogenous variables. This assumption is accepted with a p-value of 0.2. We have used MATLAB to implement the identification procedure.} By structural interpretation, we mean that it is possible to identify different shocks related to macroeconomic fundamentals, and to derive meaningful impulse response functions. In a structural VAR with long-run restrictions, the identification is achieved by positing a lower-triangular structure for the matrix of long-run impact, which requires thus to impose various long-run neutrality conditions. Since economic theory provides long-run relationships between variables, imposing long-run neutrality conditions is far more reliable than adopting contemporaneous restrictions. The focus on the long-run impact has the further advantage of filtering out temporary responses of public policies to business cycle movements. As a result, it is easier to make the distinction between genuine (fiscal and monetary) policy shocks and systematic (business cycle-related) reactions to stabilize economic activity in the short run.

Given the set of variables included in our empirical framework, we now explain how nominal bond yields can be decomposed into monetary, fiscal and supply (or productivity) shocks following the aforementioned methodology. In structural VARs, the ordering of variables matters. More precisely, in the context of long-run restrictions, the variables are put in decreasing order of long-run exogeneity. In the present case, with four endogenous I(1) variables and one cointegration relationship, we need to impose three restrictions. The first set of restrictions comes from our definition of a monetary shock. As suggested by Roberts (1993), we adopt the view that inflation is ultimately a monetary phenomenon and, accordingly, we define a monetary policy shock as a permanent shock to inflation. By using this monetarist approach, we suppose that the trend of inflation is fully under the control of the central bank. Consequently, any permanent movement in inflation results from changes in inflation that the central bank is inclined to tolerate. Inflation is thus the most exogenous variable in the long-run and explains the first two restrictions (the two zeros in the first row of Table 3).

The remaining restriction comes from our definition of a supply shock. It is widely accepted that disturbances with a permanent impact on output can be thought as aggregate supply shocks (often referred as technology or productivity shocks). Consequently, only supply shocks—beyond the
monetary shocks\(^1\)—can have a permanent effect on output and thus explain the third restriction (the zero in the second row of Table 3). Finally, the fiscal policy shock is an exogenous, permanent disturbance to the primary fiscal balance (expressed as a percentage of GDP).\(^2\) This deficit measure has three main advantages. First, it is representative of the government’s financial needs and, thereby, is more likely to have an impact on bond yields than a fiscal measure based on taxes or government expenditures only. Second, by defining a fiscal shock in term of its long-run impact, we implicitly assume that the fiscal shock is purged of any business cycle movements. Third, since the shocks we consider are permanent, our approach is consistent with the more and more widely accepted view that fiscal policy can impact interest rates only if it is considered permanent or structural, as opposed to a temporary fiscal change.

4. Shock Analysis

The long-run impact of the three structural shocks is displayed in Table 3.

Table 3: Long-run matrix of the structural shocks

<table>
<thead>
<tr>
<th></th>
<th>(\eta^n)</th>
<th>(\eta^y)</th>
<th>(\eta^f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inf</td>
<td>0.66</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>gdp</td>
<td>1.05</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>def</td>
<td>0.5</td>
<td>-0.03</td>
<td>0.17</td>
</tr>
<tr>
<td>r</td>
<td>0.41</td>
<td>0.02</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The typical nominal shock increases inflation by around 0.7 percent in the long-run while the real long-term interest rate decreases both in the short and the long run, violating the Fisher effect (see Figure 3). The permanent decrease in the real interest rate is consistent with King and Watson (1997) who show that, in a large set of identification schemes, nominal interest rates do not fully adjust to permanent inflation shocks.\(^3\) This is also in line with Rapach (1999) - who find that the

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1. This identification structure allows for the possibility that monetary superneutrality would not hold in the long run. Such a scenario is consistent with the view that inflation could have distortionary effects on real output.
2. Assessing the potential impact of fiscal policy on long-term interest rates effectively requires to remove the component of fiscal policy that is explained by interest rates: the interest paid on the public debt.
3. Notice how the impact of the (structural) nominal shock is at odd with the (false) interpretation of the cointegrating vector coefficients (see footnote 4).
real interest rate decreases following a permanent increase in inflation for all the 14 industrialised countries studied - and with Gauthier and Li (2004) in the context of a larger model for Canada. This result, however, is at odd with Mehra (1996) who concludes in favor of a cointegration relationship between inflation and the long rates in the U.S., thus implying that long-term interest rates are explained solely by inflation in the long -run.

The level of real output is affected both in the short and long run by a persistent increase in inflation. The short run increase is consistent with the accomodative lower real interest rate. The empirical evidence regarding the long-run impact of inflation on real output, however, is mixed. For example, King and Watson (1997) conclude that superneutrality with respect to output can be rejected for some identification schemes that they consider reasonable. They also find that the effect can be either way.¹ Bullard and Keating (1995) also document that permanent inflation shocks permanently increase the level of output for certain low inflation countries.

The typical supply shock (see Figure 4) increases output by around 1.2 percent in the long-run. In conformity with most theoretical models, an unexpected permanent rise in output increases excess supply and pushes inflation down in the short-run. The fiscal deficit is decreased in the short term as higher income brings higher government revenues and lower transfers. The lower real interest rate in the short run suggests that the Bank of Canada has historically accomodated supply shocks. It is interesting to note that the real interest rate long-run value is unaffected by supply (technology) shocks. This is consistent with the model of Ramsey in which the interest rate is determined by the rate of time preferences while the level of capital is set by technology so that the marginal product of capital is equal to the interest rate. These results are in line with Gauthier and Li (2004).

The impact of a permanent increase in the government primary deficit is reported in Figure 5. As predicted by standard macroeconomic models, a positive fiscal shock (coming from either an increase in spending or a decrease in taxes) stimulates the economy and slightly increases inflation in the short-run as excess demand is built. Our results suggest that along with deficit increases in Canada, important risk premiums were incorporated in the long-term interest rate. More precisely, a permanent unexpected deficit increases of around 0.20 percent is associated with a 40 basis points rise in the long-term interest rate. Such a strong influence of fiscal policy on Canadian long-term interest rates has already been documented in the existing empirical literature. However, comparing our results with those provided by this literature is delicate given

¹ There exist theories that are consistent with both possibilities.
the differences in both variables and methodologies. For example, Fillion (1996) suggests that the increase in the public sector debt ratio from 1990 to 1994 accounts for an increase of 85 to 135 basis points in real interest rates. Not only does Fillion examine the impact of an increase in the debt ratio while we focus on a deficit measure, but his results are based on the long-run coefficients in the cointegrating relationship while our results are based on the impact of structural shocks which may differ considerably from the reduced form coefficients (see footnotes 4 and 9).¹ Moreover, our data span 40 years, twice longer than Fillion’s sample (1975-1994). Nevertheless, both papers conclude that fiscal policy is an important long-run determinant of long-term interest rates.

The key role played by the fiscal policy stance regarding long-term interest rates is clearly illustrated by the historical components (Figure 6). The important deficit reduction started during the mid-eighties is by far the most important factor explaining the decrease in long-term nominal interest rates over the following years, explaining more than half of it. Symmetrically, the important fiscal deterioration observed from the mid-seventies to the mid-eighties explain the biggest part of the strong increase in interest rates over the same period. Inflation shocks were also an important determinant of interest rates. The impact of supply shocks is more mitigated. Still, the investment boom of the nineties is responsible for a 200 basis points decrease in long-term interest rates.

5. Conclusion

This paper proposes to relate the dynamics of nominal bond yields to various macroeconomic drivers. Based on existing literature, monetary policy, fiscal policy and supply shocks are relevant candidates as determinants of the long-term nominal interest rate. Using a structural VECM that includes one exogenous variable, we study the dynamics of Canadian long-term nominal bond yields on the 1962-2003 period.

We show that fiscal policy is a key determinant of interest rates: a permanent unexpected deficit increases of around 0.20 percent creates a 40 basis points rise in the long-term interest rate. Moreover, we confirm the impact of monetary policy on long-term nominal interest rates through their inflationary component. A one per cent permanent unexpected rise in inflation increases the long-term nominal interest by around 0.6 percent in the long-run.

¹ Fillion estimates one model including both the debt ratio and the deficit ratio, but this model is mis-specified if, as appears to be the case, both variables are treated as being integrated of the same order.
In future research, we plan to extend our empirical study to the U.S. case. Effectively, in light of the recent deterioration of the U.S. public finances, it would be particularly interesting to replicate this study with U.S. data to consider whether the current situation may impact U.S. long-term nominal interest rates. Finally, we will also include the neutral interest rate coming out of our model - defined as its long-run (equilibrium) value - in a future version of this paper.
REFERENCES


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Figure 1. Inflation rate and Government of Canada 10-year Bond Yields
Figure 2. The Data
Figure 3. Impact of a typical permanent inflation shock
Figure 4. Impact of a typical supply shock

- Supply shock on inflation
- Supply shock on output
- Supply shock on deficit
- Supply shock on interest rate
Figure 5. Impact of a typical deficit shock
Figure 6. Historical components of the long-term interest rate
Appendix A. Identification of shocks in a VECM with exogenous variables

In a non-cointegrated VAR model, the structural shocks’ identification procedure (Blanchard and Quah [1989] for example) is clearly invariant to the presence or not of exogenous variables in the model. However, in presence of cointegration, this is not obvious as the common stochastic trends must be consistent with the cointegrating relations which possibly include exogenous variables. Wickens and Motto (2001) has shown how to identify the shocks when the following restrictions are made: the variables can be classified as endogenous or exogenous, there are as many cointegrating relations as endogenous variables, the cointegrated vectors are identified and they contain at least one exogenous variable. In Wickens and Motto (2001) the complete model need to be estimated. In this paper, we show how King et al. (1991)’s identification procedure can be applied to a VECM with either weakly exogenous I(1) variables restricted not to be in the cointegrating relations, or strongly exogenous variables. A simple way to invert a VECM with exogenous variables is also suggested.

A.1 Efficient estimation of a VECM with weakly exogenous variables

Economic systems often have so many potentially useful variables that the system gets extremely large. Johansen (1992) has shown, however, that a partial model can be efficiently estimated when some of the variables are weakly exogenous. Consider an m-dimensional VAR(p) process \( \{ z_t \}_{t=1}^{\infty} \) expressed as the vector error correction model (VECM):

\[
\Delta z_t = a + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + e_t, \quad t=1,2,\ldots \tag{A.1}
\]

where \( \Delta = 1 - L \) with \( L \) being the lag-operator, the long-run multiplier \( \Pi \) and the short-run response matrices \( \Gamma_i \) are \( m \times m \) constant coefficient matrices, \( a \) is a constant vector, and the \( m \)-dimensional disturbance \( e_t \sim IN(0, \Omega) \).

We now partition the \( m \)-vector of random variables \( z_t \) into the \( n \)-vector \( y_t \) and the \( k \)-vector \( x_t \), where \( k = m - n \); that is, \( z_t = (y_t', x_t')' \), \( t = 1, 2, \ldots \). By partitioning the error term \( e_t \) conformably with \( z_t = (y_t', x_t')' \) as \( e_t = (e_{y_t}', e_{x_t}')' \) and its variance matrix as
we are able to express $e_{yt}$ conditionally in terms of $e_{xt}$ as

$$e_{yt} = \Omega_{yx} \Omega_{xx}^{-1} e_{xt} + u_t,$$

where $u_t \sim IN(0, \Omega_{uu})$, $\Omega_{uu} = \Omega_{yy} - \Omega_{yx} \Omega_{xx}^{-1} \Omega_{xy}$ and $u_t$ is independent of $e_{xt}$. We also use a similar partitioning of the parameter vectors and matrices $a = (a'_y, a'_x)'$, $\Pi = (\Pi'_y, \Pi'_x)'$ and $\Gamma_i = (\Gamma'_{yi}, \Gamma'_{xi})'$, $i = 1, \ldots, p - 1$. Following Johansen (1992) and Boswijk (1992, Chapter 3), we make the following assumption:

**Assumption 2.1** $\Pi_x = 0$.

Under Assumption 2.1, i.e. the process $\{x_t\}_{t=1}^{\infty}$ is weakly exogenous with respect to the matrix of long-run multiplier $\Pi$, the following conditional model in terms of $z_{t-1}, \Delta x_t, \Delta z_{t-1}, \Delta z_{t-2}, \ldots$ is efficiently estimated by maximum likelihood without using the equations for $\{x_t\}_{t=1}^{\infty}$:

$$\Delta y_t = c + \Lambda \Delta x_t + \sum_{i=1}^{p-1} \psi_i \Delta z_{t-i} + \Pi y z_{t-1} + u_t, \quad t = 1, 2, \ldots \quad (A.3)$$

where $c \equiv a_y - \Omega_{yx} \Omega_{xx}^{-1} a_x$, $\Lambda \equiv \Omega_{yy} \Omega_{xx}^{-1}$, and $\psi_i \equiv \Gamma_{yi} - \Omega_{yx} \Omega_{xx}^{-1} \Gamma_{xi}$, $i = 1, \ldots, p - 1$.

### A.2. Identification of the permanent shocks

The identifying procedure documented in King *et al.* (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. An easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example) is proposed in Appendix B. The inverted reduced form model obtained is:

$$\Delta y_t = \mu + C_x(L) \Delta x_t + C(L) u_t \quad (A.4)$$

where all the parameters are defined in Appendix A. Notice that, since $u_t$ is independent of $e_{xt}$, $u_t$ is independent of $\Delta x_t$.

Consider a structural model of the form:

$$\Delta y_t = \mu + C_x(L) \Delta x_t + \Gamma(L) \eta_t \quad (A.5)$$
where \( \eta_t \sim \mathcal{N}(0, \Omega_\eta) \) is a \( n \times 1 \) vector of serially uncorrelated disturbances independent of \( \Delta x_t \) (being a linear combination of \( u_t \)), and where the endogenous variables’ response to a change in the exogenous variables is given by \( C_x(L) \).

The identifying problem consist in identifying the individual components in \( \eta_t \) from the estimated reduced form model given by (4) and can be described as follows. There are \( s = n - r \) identifiable common stochastic trends driving the \( n \times 1 \) vector \( y_t \) where \( r = \text{Rank}[\Pi_y] \).\(^1\) We express \( \Pi_y = \alpha_y \beta' \) where the \( n \times r \) loading matrix \( \alpha_y \), and the \( m \times r \) matrix of cointegrating vector \( \beta \) are each full column rank and identified up to an arbitrary \( r \times r \) non-singular matrix.\(^2\) Partition \( \beta \) conformably with \( z_t \) as \( \beta' = (\beta_y', \beta_x')' \) where \( \beta_y \) and \( \beta_x \) are respectively \( n \times r \) and \( k \times r \), and partition the vector of structural disturbances \( \eta_t \) into two components, \( (\eta_t^1', \eta_t^2')' \), where \( \eta_t^1 \) contains the \( s \) disturbances that have permanent effects on the components of \( y_t \) and where \( \eta_t^2 \) contains \( n - s \) elements that have only temporary effects.

Partition the matrix of long-run multipliers, \( \Gamma(1) \), conformably with \( \eta_t \) as \( \Gamma(1) = [\Theta, 0] \), where \( \Theta \) is the \( n \times s \) matrix of long-run multipliers for \( \eta_t^1 \) and \( 0 \) is a \( n \times (n - s) \) matrix of zeros corresponding to the long-run multipliers of \( \eta_t^2 \).

**Assumption 3.1** \( \beta_x' = 0 \)

Under Assumption 3.1, \( \beta'z_t \) being stationary implies that \( \beta'y_t \) is stationary, which implies \( \beta'y_t \Gamma(1) = 0 \). Hence the matrix of long-run multipliers is determined by the condition that its columns are orthogonal to \( \beta_y' \), and \( \Theta \eta_t^1 \) represents the innovations in the long-run components of \( y_t \). While the cointegration restrictions identify the permanent innovations \( \Theta \eta_t^1 \), they fail to identify \( \eta_t^1 \) because \( \Theta \eta_t^1 = (\Theta P)(P^{-1} \eta_t^1) \) for any non-singular matrix \( P \). To identify the individual elements of \( \eta_t^1 \), we need the following identifying restrictions:

**Assumption 3.2.** \( u_t = \Gamma_0 \eta_t \) where \( \Gamma_0^{-1} \) exists.

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\(^1\) We implicitly make the assumption that \( s \) is strictly positive. Wickens (1996) has shown that if \( \text{rank} [\Pi] = n \), then the full model has to be estimated and the common stochastic trends can be equated with the non-stationary component of the exogenous variables.

\(^2\) That is, \( (\alpha, K^{-1})(K\beta') = (\alpha, \beta') \) for any \( (r, r) \) non-singular matrix \( K \).
Under assumption 3.2, the structural disturbances are in the space spanned by current and lagged values of $z_t$, and that there are no singularities in the structural model.

**Assumption 3.3.** $\Theta$ is assumed triangular which permits writing $\Gamma(1) = [\tilde{\Theta}\Pi, 0]$ where $\tilde{\Theta}$ is a $n \times s$ matrix with no unknown parameters whose columns are orthogonal to $\beta_y'$, and $\Pi$ is a $s \times s$ lower triangular matrix with full rank and 1's on the diagonal.$^1$

The covariance matrix of the structural disturbances is partitioned conformably with $\eta_t = (\eta_t^1, \eta_t^2)'$ and is assumed to be

**Assumption 3.4.** $\Omega_{\eta} = \begin{bmatrix} \Omega_{\eta^1} & 0 \\ 0 & \Omega_{\eta^2} \end{bmatrix}$ where $\Omega_{\eta^1}$ is diagonal.

That is, the permanent shocks, $\eta_t^1$, are assumed to be uncorrelated with the transitory shocks, $\eta_t^2$, and the permanent shocks are assumed to be mutually uncorrelated.

The permanent innovations, $\eta_t^1$, can be determined from the reduced form (7) as follows. From equations (7) and (8) and Assumption 3.2, $C(L) = \Gamma(L)\Gamma_0^{-1}$ and $C(1) = \Gamma(1)\Gamma_0^{-1}$. Let $D$ be any solution of $C(1) = \tilde{\Theta}D$. Thus, $\tilde{\Theta}Du_t = \tilde{\Theta}\Pi\eta_t^1$ and $D\Omega_uD' = \Pi\Omega_{\eta}\Pi'$. Let $\Pi = chol(D\Omega_uD') = \Pi\Omega_{\eta}^{1/2}$. Since $\Pi$ is a triangular matrix, and $\Omega_{\eta^1}$ is diagonal, there is a unique solution for $\Pi$ and $\Omega_{\eta^1}$. We can thus identify the permanent shocks $\eta_t^1 = \Pi^{-1}Du_t$. Defining $G = \Pi^{-1}D$, it is then easy to show that the dynamic multipliers associated with $\eta_t^1$ are $C(L)\Omega_uG\Omega_{\eta}^{-1}$.

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1. The diagonal elements of $\Pi$ are normalised to unity without loss of generality, since the variances of $\eta_t^1$ in Assumption 4.3 are unrestricted.
Appendix B. A simple way to invert a VECM with exogenous variables.

The identifying procedure documented in King et al. (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. In this section, we propose an easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example).

By partitioning $\Pi_y$ and $\Psi_i$ conformably with $z_t = (y'_t, x'_t)'$ as $\Pi_y = (\Pi_y^x, \Pi_y^y)'$ and $\Psi_i = (\psi_i^y, \psi_i^x)'$, where $\Pi_y^x$ and $\psi_i^y$ are $n \times n$ and $\Pi_y^y$ and $\psi_i^x$ are $n \times k$ constant coefficient matrices, we can rewrite (A.3) as:

$$ y_t = c + B_0 x_t + \sum_{i=1}^{p} A_i y_{t-i} + \sum_{i=1}^{p} B_i x_{t-i} + u_t $$  \hspace{1cm} (B.1)

where $B_0 = \Lambda$, $B_1 = - (\Lambda - \Pi_y^x - \psi_1^y)$, $B_i = (\psi_i^y - \psi_{i-1}^y)$ for $i = 2, \ldots, p - 1$, $B_p = - \psi_{p-1}^x$, $A_1 = (\psi_1^y + \Pi_y^y + I_n)$, $A_i = (\psi_i^y - \psi_{i-1}^y)$ for $i = 2, \ldots, p - 1$ and $A_p = - \psi_{p-1}^y$.

We then write (A.4) as the following VARX(1):

$$ y_t = C + Ay_{t-1} + Bx_t + U_t $$  \hspace{1cm} (B.2)

where $y_t = (y'_t, y'_{t-1}, \ldots, y'_{t-p+1}, x'_t, x'_{t-1}, \ldots, x'_{t-p+1})'$, $U_t = (u'_t, 0, 0, \ldots, 0)'$ and $C = (c', 0, 0, \ldots, 0)'$ are $mp \times 1$ matrices. Matrices $A$ and $B$, respectively of dimension $mp \times mp$ and $mp \times k$, are defined accordingly to $Y$ and $x$ following Lutkepohl (p.335). Assuming that the process starts at a finite time $t = 0$, it is straightforward to obtain the inverted form:

$$ y_t = A'^t y_0 + \sum_{i=0}^{t-1} A'^i c + \sum_{i=0}^{t-1} A'^i B x_{t-i} + \sum_{i=0}^{t-1} A'^i U_{t-i} $$  \hspace{1cm} (B.3)

Taking the first difference of (B.3), assuming for simplicity that $U_0 = x_0 = y_0 = 0$, and extracting the endogenous variables with the appropriate $nm \times p$ matrix $J = [I_n', 0, \ldots, 0]'$, we get:

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1. In this unstable system, a one time impulse may have a permanent effect in the sense that it shifts the system to a new equilibrium, but the impulse responses may be calculated just as in the stable case. See Lutkepohl, Reimers (1992) for further details on this point.
\[ \Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t \]  \hspace{1cm} \text{(B.4)}

where \( \mu = JA^{t-1}c \), \( C_x(L) = \sum_{i=0}^{t-1} JA^i B L^i \), \( C(L) = \sum_{i=0}^{t-1} C_i L^i \),

\[ C_i = J(A^i - A^{i-1})J' L^i \] for \( i = 1, \ldots, t-1 \) and \( C_0 = I_n \).